ADOPTING AN OPTIMAL MAINTENANCE STRATEGY BY USING MULTICRITERIAL DECISIONAL MODELS

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Abstract - The paper aims to underlie an optimal maintenance policy by solving a "case study".
The technical system that serves as "case study" consists of three electrical driven pumps having the operational logical majority regime of "2 out of 3"
In order to underlie the optimal solution, the following methods are used: the entropy method, ELECTRE-Boldur method and the method of aggregation function.

Key words: operational logical majority regime, aggregation function, entropy.

1. INTRODUCTION

The paper presents three multicriterial decisional models:

- the entropy method: used in order to achieve a mediation between the entropic effect corresponding to a certain criterion and the relatively subjective proposals regarding the "importance" of that criterion;
- the aggregation function method: aims to obtain the optimal compatibility of locations (hierarchies) and alternatives – criteria;
- Electre-Boldur method: used for underlying the optimal order taking the hypothesis of accepting – exclusively – the decidents opinions on their criteria.

It may be observed that the same optimal order of alternatives results (defined by the convergence of all three methods).

2. CASE STUDY

A group of three electrical driven pumps, belonging to an urban district heating network, is considered. From a structural point of view, the equipment is identical, but shows different reliability levels:

$R(EP_1) = 0.90; R(EP_2) = 0.80; R(EP_3) = 0.95$

Rated operational regime is redundant logical majority, type "2+1".

The group of pumps has a rather high reliability: R (S) = 0.967, but the decident factor takes into consideration the following decisional alternatives:

V_1 – do not take any action;

V_2 – repair EP_2 electrical pump, since it has the lowest reliability level;

V_3 – get an equipment from another unit that has on stock a recently revised pump with an estimated reliability of $R'(EP_2) = 0.851$;

V_4 – replace EP_2 electrical pump.

Decisional criteria proposed by the decident factor are:

C_1 – the cost associated to each alternative;

C_2 – the maintenance period (time extent);

C_3 – the reliability of the group of equipment corresponding to each alternative.

Table 1 shows the above mentioned alternatives, criteria and decisional consequences corresponding to each (C_i, V_j) couple.

Table 1

C_i / V_j	C_1	C_2	C_3
V_1	35k	20k'	0,9670
V_2	50k	30 k'	0,9838
V_3	40k	10 k'	0,9742
V_4	100k	50 k'	0,9923

k, k' are the multiplication factors

Observations:

1. Costs associated to V_1 alternative are due to regular maintenance works.
2. Moreover, the period corresponding to this first alternative (V_1) considers the necessary time for preventive maintenance, while for V_3 alternative (having a shorter time period) it is taken into account that - once the pump is replaced - the weight of preventive checkings is diminished. This is due to the fact that the replacement of the broken pump doesn't need a significant period of time.
3. As a result of maintenance works applied to electrical pump 2 (alternative V_2), the reliability of this pump, R_2=0,80, will increase with 15 %, reaching $R_2' = 0.92$

Regarding the reliability of V_3 alternative, it includes a higher level of reliability (corresponding to the replacement pump) as compared to EP_2:

$R'_{replacement\ pump} = 0.851 > R_2 = 0.80$;

therefore, a gain of approx. 6 %.

In case of replacing EP_2 electrical pump with another one having a reliability of $R^* = 0.98$ (value provided by the manufacturer), the reliability level of the system improves with approx. 2.65 %, reaching R(S) = 0.9922. Table 2 presents the utilities associated to decisional consequences.

Table 2

C_i /V_j	C_1	C_2	C_3
V_1	1,000	0,75	0
V_2	0,769	0,50	0,664
V_3	0,923	1,00	0,285
V_4	0	0	1,000

In order to calculate U_{ij} utilities, the following relations were used:

$$U_{ij} = \frac{A^{\max} - A_{ij}}{A^{\max} - A^{\min}} \qquad (1)$$

OPTIMUM $\Rightarrow$ MINIMUM

$$U_{ij} = \frac{A_{ij} - A^{\min}}{A^{\max} - A^{\min}} \qquad (2)$$

OPTIMUM $\Rightarrow$ MAXIMUM

A^{max}, A^{min}, A_{ij} represents the decisional consequences

$I = \overline{1,n}$; $j = \overline{1,m}$;

n – number of criteria

m – number of alternatives

A coefficient of importance, k_i, was associated to each criterion "i". For this, the decident group proposed the values presented in the below table, where k^*_i are the normalized coefficients of performance, calculated according to relation (3).

$$k_i^* = \frac{k_i}{\sum_i k_i} \qquad (3)$$

Table 3

C_i	C_1	C_2	C_3
k_i	5	6	10
k_i^*	0,238	0,286	0,476

3. THE ENTROPY METHOD

The next steps must be followed:

a. Normalized utilities are determined using relation (4):

$$U_{ij}^* = \frac{U_{ij}}{\sum_j U_{ij}} \qquad (4)$$

b. For each C_i criterion, the Shannen way entropy is deduced using relation (5):

$$H_i = -\frac{1}{\ln m} \sum p_{ij} \ln p_{ij} \qquad (5)$$

where p_{ij} probabilities are equivalent to normalized utilities U_{ij};

c. A parameter called degree of information diversification d_i – complementar to entropy H_i – is calculated with (6)

$$d_i = 1 - H_i \qquad (6)$$

d. Entropic coefficients of performance are calculated using (7)

$$k_i^H = \frac{d_i}{\sum_i d_i} \qquad (7)$$

e. Weighted coefficients of performance are determined with (8)

$$k_i^\circ = \frac{k_i^* \cdot k_i^H}{\sum k_i^* \cdot k_i^H} \qquad (8)$$

A summary of the results obtained from the above formulas is presented in Table 4.

Table 4

	V_j\C_i	C_1	C_2	C_3	
U_{ij}	V_1	1,000	0,75	0	
	V_2	0,769	0,50	0,664	
	V_3	0,923	1,00	0,285	
	V_4	0	0	1,000	
$\sum_j U_{ij}$		2,692	2,25	1,949	
U^*_{ij}	V_1	0,371	0,333	0	
	V_2	0,286	0,222	0,341	
	V_3	0,343	0,445	0,146	
	V_4	0	0	0,513	
$\sum_j U^*_{ij}$		1,000	1,000	1,000	
H_i		0,78835	0,76533	0,71429	$\sum_i$
d_i		0,2116	0,234	0,2857	0,7320
k_i^H		0,289	0,321	0,390	1,000
k_i^*		0,238	0,286	0,476	1,000
$k_i^* \cdot k_i^H$		0,069	0,092	0,186	0,347
k_i°		0,199	0,265	0,536	1,000

Table 5 shows the calculation elements in order to get the differentiation of alternatives.

Table 5

k_i°	0,199	0,265	0,536	$\sum_i k_i^\circ \cdot U_{ij}$
V_j \ C_i	C_1	C_2	C_3	
V_1	0,199	0,199	0	0,398
V_2	0,153	0,133	0,356	0,642
V_3	0,183	0,265	0,153	0,601
V_4	0	0	0,536	0,536

Analyzing the data contained in the last column of Table 5, the following succession (sequence) was determined:

V2 $\mathcal{P}$ V3 $\mathcal{P}$ V4 $\mathcal{P}$ V1

where "$\mathcal{P}$" is the preferential symbol.
Therefore, the optimal alternative is the second one: TO REPAIR the electrical pump EP_2. The optimization criterion is given by the following relation:

$$\max_j \sum_i k_i^\circ \cdot U_{ij} \tag{9}$$

If the calculations are not done based on entropy, it is possible to rise situations of undecidability.
Indeed, if the utilities U_{ij} are kept, but the normalized coefficients of performance are used, k_i^*, the following values are obtained (Table 6).

Table 6

k_i°	0,238	0,286	0,476	$\sum_i k_i^\circ \cdot U_{ij}$
$V_j \setminus C_i$	C_1	C_2	C_3	
V_1	0,238	0,215	0	0,453
V_2	0,183	0,143	0,316	0,642
V_3	0,220	0,286	0,136	0,642
V_4	0	0	0,476	0,476

The next sequence is obtained:

V2 $\mathcal{I}$ V3 $\mathcal{P}$ V4 $\mathcal{P}$ V1

where „$\mathcal{I}$" marks the state of undecidability.

4. THE METHOD OF AGGREGATION FUNCTION

To apply this method, the next steps must be followed:

a. The matrix L of places corresponding to C_i criterion and V_j alternative is written as follows:

$$L = \begin{matrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_m \end{matrix} \begin{bmatrix} L_{11} & L_{21} & . & . & L_{n1} \\ L_{12} & L_{22} & . & . & L_{n2} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ L_{1m} & L_{2m} & . & . & L_{nm} \end{bmatrix}. \tag{10}$$

b. The appreciation function is defined:

$$A(V_j) = \sum_i \left[n - loc(C_i, V_j)\right] \cdot k_i^* \tag{11}$$

c. Diameter function is defined:

$$d(V_j) = \max_i \left[loc(C_i, V_j)\right] - \min_i \left[loc(C_k, V_l)\right] \quad i,k = \overline{1;n};\ j,l = \overline{1;m} \tag{12}$$

d. Aggregation function is made:

$$A_{ggr}(V_j) = \left\{A(V_j) + \left[n - d(V_j)\right]\right\}/2 \tag{13}$$

By introducing the calculation data, the parameter values needed for alternatives ranking, are successively deduced:

$$L = \begin{matrix} \\ loc_1 \\ loc_2 \\ loc_3 \\ loc_4 \end{matrix} \begin{matrix} C_1 & C_2 & C_3 \\ V_1 & V_3 & V_4 \\ V_3 & V_1 & V_2 \\ V_2 & V_2 & V_3 \\ V_4 & V_4 & V_1 \end{matrix}$$

$A(V_1)=(3-1)0,238+(3-2)0,286+(3-4)0,476=0,286;$
$A(V_2)=(3-3)0,238+(3-3)0,286+(3-2)0,476=0,476;$
$A(V_3)=(3-2)0,238+(3-1)0,286+(3-3)0,476=0,810;$
$A(V_4)=(3-4)0,238+(3-4)0,286+(3-1)0,476=0,428;$
$d(V_1)=4-1=3;$
$d(V_2)=3-2=1;$
$d(V_3)=3-1=2;$
$d(V_4)=4-1=3;$
$A_{ggr}(V_1)=[0,286+(3-3)]/2=0,143;$
$A_{ggr}(V_2)=[0,476+(3-1)]/2=1,238;$
$A_{ggr}(V_3)=[0,810+(3-2)]/2=0,905;$
$A_{ggr}(V_4)=[0,428+(3-3)]/2=0,214$

The optimum criterium is:

$$\max_j A_{ggr}(V_j) \Rightarrow \text{optimal alternative} \tag{14}$$

Therefore, the next sequence results:

V2 $\mathcal{P}$ V3 $\mathcal{P}$ V4 $\mathcal{P}$ V1

It must be noticed that resulted the same sequence for ranking the alternatives.

5. ELECTRE – BOLDUR METHOD

This calculation technique is a version of ELECTRE method proposed by professor Gh. Boldur – Lățescu.
The steps are the following:

a. Previous determined parameters – normalized coefficients of performance (k^*_i) and respective utilities U_{ij} – are kept;
b. Coefficients of concordance $c^*(V_j;V_l)$ and coefficients of discordance $d^*(V_j;V_l)$ are determined:

$$c^*(V_j, V_l) = \sum_i (U_j - U_l) \cdot k_i^* \tag{15}$$

$$d^*(V_j, V_l) = c^*(V_l, V_j) \tag{16}$$

c. Matrices associated to these indicators are built

$$M(C^*(V_{j,}, V_l)), M(d^*(V_j, V_l))$$

d. For each line of matrix $M(c^*(V_j;V_l))$ the minimum value of concordance indicator, respective the maximum value of discordance indicator for matrix $M(d^*(V_j;V_l))$ are determined

e. Calculation of differences

$$\Delta \equiv \max_j(c^*(V_j,V_l)) - \min_j(d^*(V_j,V_l)) \qquad (17)$$

f. Setting the ranking of alternatives according to the optimization criterion :

$$\max \Delta \Rightarrow \text{optimal ranking} \qquad (18)$$

For the given calculation data, the following values result:

$c^*(V_1,V_2)=0,238(1-0,769)+0,286(0,75-0,5) \Rightarrow 0,1265;$
$c^*(V_1, V_3)=0,238(1-0,923) \Rightarrow 0,0183;$
$c^*(V_1,V_4)=0,238(1-0)+0,286(0,75-0) \Rightarrow 0,4525;$
$c^*(V_2,V_1)=0,476(0,664-0) \Rightarrow 0,3161;$
$c^*(V_2,V_3)=0,476(0,664-0,285) \Rightarrow 0,1804;$
$c^*(V_4,V_3)=0,476(1- 0,285) \Rightarrow 0,3403;$

Indicators $d^*(V_j;V_l)$ are written based on relation (16): thus elements belonging to lines of matrix $M(c^*(V_j;V_l))$ become elements of columns of $M(d^*(V_j;V_l)$ matrix.

The matrices associated to these indicators are:

$\|c^*(V_j,V_l)\| =$

	V_1	V_2	V_3	V_4		minimum
V_1		0,0183	0,0183	0,4525	$\Rightarrow$	0,0183
V_2	0,3161		0,1804	0,3260	$\Rightarrow$	0,1804
V_3	0,2072	0,1796		0,5057	$\Rightarrow$	0,1796
V_4	0,4760	0,1600	0,3403		$\Rightarrow$	0,1600

$\|d^*(V_j,V_l)\| =$

					maximum
	0,3161	0,2072	0,4760	$\Rightarrow$	0,4760
0,1265		0,1796	0,1600	$\Rightarrow$	0,1796
0,0183	0,1804		0,3403	$\Rightarrow$	0,3403
0,4525	0,3260	0,5057		$\Rightarrow$	0,5057

$$\Delta = \max_j\left[(0,0183-0,4760);(0,1804-0,1796);(0,1796-0,3403);(0,1600-0,5057)\right]$$

Sau

$$\Delta = \max_j(-0,4577 \in V_1;+0,008 \in V_2;-0,1607 \in V_3;-0,3457 \in V_4)$$

According to the optimization criterion (18) the following sequence is obtained:

V2 𝒫 V3 𝒫 V4 𝒫 V1

In can be seen that the output is the same as in the two previous methods.

CONCLUSIONS

- Multicriterial decisional methods presented above converge to the same optimum solution. But this is not really necessary in practice. The decident will prefer a unique way calculation.
- The subjective element of all methods of underlie the optimal decision is setting the weight of coefficients of importance, because their values greatly depend on the members of decident group. The entropy method gives a certain "dilution" of this "subjectivism" as a result of introducing the diversification degree of information (a complementary parameter to informational entropy).

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