POWER GRIDS' ROBUSTNESS – ORIENTED ENLARGEMENT TOOLS

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Abstract - The safety of power grids can be formulated in several ways: the most familiar is to consider the failure processes as inherent stochastic events in complex systems. In addition, we accept the failure processes affect a very limited area of the studied network. This approach is important in the design in the networks extension with new nodes due to new energy users. On the contrary, the vulnerability express the systems' behavior opposite to the disturbances generated by natural disasters, adverse weather, technical failures, human errors, sabotage, terrorism, and acts of war, thus scenarios within large areas are damaged. In this approach, the robustness is the network's capacity to support random destroying experiences. Today, one general accepted approach in the vulnerability quantification is made in terms of scale-free network, because these particular networks have a high capability to conserve their robustness. Our paper proposes some new analytical tools in the enlargement of the electrical distribution networks, the goal being to achieve, or conserve a desirable robustness level. In this line, the magnitude of the robustness is one of the energy distribution grids risk management targets. The main part of our study try to formulate useful methods in optimal adjustments, of non-robust networks to the nearest free-scale, robust network, continuing our earlier studies exposed in [2].

Key words: networks' vulnerability, scale free networks, critical threshold fraction

1. INTRODUCTION

The main elements to describe large systems by their graphs are presented in [2]. We'll reproduce here only the quantifiers which will be developed in the present work.

Generally, networks considered as graphs can be characterized by a number of parameters:

- The nod's degree k, which tells how many links the node has to other nodes.
- The degree distribution *P(k)* representing the probability that a selected node has exactly k links. *P(k)* is obtained by counting the number of nodes *N(k)* with *k = 1, 2...* links and dividing by the total number of nodes n. Hence, the average degree will be:

$$<k> = \sum_k k \cdot P(k) \qquad (1)$$

- The critical fraction *fc* quantifies the networks' robustness, visa random series of removed nodes (random attacks) are generated: after every node deletion, a maximal cluster dimension S, depending to the fraction of deleted nodes is kept. Graphically, there is a point corresponding to the critical fraction, from where the decreasing of the remaining clusters dimension changes its slope.
- One computable measure of the critical threshold *fc* for random fragmentation was proposed by Cohen et al. (2000). a criterion for *fc* is realized when [1 Cohen]:

$$\frac{<k^2>}{<k>} = 2 \qquad (2)$$

where:

$$<k^2> = \sum_k k^2 \cdot P(k) \qquad (3)$$

Hence a useful formula permeates to calculate the critical fraction f_c for one ordinary graph described by its probability function $\begin{pmatrix} k \\ P(k) \end{pmatrix}$ is:

$$f_c = 1 - \frac{1}{\frac{<k^2>}{<k>} - 1} \qquad (4)$$

For scale-free networks with the degree distribution:

$$P(k) = Ak^{-b}, \quad k = m, m+1, ..., K \qquad (5)$$

the value of $\frac{<k^2>}{<k>}$ is approximated with:

$$\frac{<k^2>}{<k>} = \begin{cases} m^{b-2} K^{3-b} & if\ 2 < b < 3 \\ K & if\ 1 < b < 2 \end{cases} \qquad (6)$$

for $2 < b < 3$, a practical approximation for the critical threshold f_c can be used [2]:

$$f_c = 1 - \frac{1}{2b-1} \quad (7)$$

2. ADJUSTING NON-SCALE FREE NETWORKS TO THE NEARLY SCALE-FREE NETWORK

Let done one a NT_0 network with the empiric degree distribution *P(k)*. We assume that the network adjustment will be realized by adding only new edges to the existing connections, without adding new nodes.

In a logarithmic coordinates system, log(P(k)) and log(N(k)) define lines:

$$\log(P(k)) = A - b\cdot\log(k), \log(N(k)) = A_1 - b\cdot\log(k) \quad (8)$$

a nearest line will funded by linear regression method, representing a theoretical (Eq.8)- form degree distribution function, characterized by the values of *A* and *b*. The problem consists in getting $\begin{pmatrix} k \\ P^F(k) \end{pmatrix}$, so that for all integer k's to verify the existence of one graph N^F , with the same nodes' number $\sum N(k)$ as the initial, non-scale-free network, with the rational demand to assure a certain minimal critical fraction f_c. According with (Eq.7) and (Eq.8), to achieve a fixed f_c is equivalent to search, in logarithmic coordinates, a line having as minimal slope *S*, so that $tgS = -b$ .Let consider as solved this problem.

Thus we must to obtain the objective graph G_F, from the initial G_0 graph of the studied power grid. In other words, we'll have to identify the elementary transformations from G_0 to G_F .We decide to choose for adjustments, only the nodes with $N_k^0 < N_k^F$, considering that in the contrary case , the additional connections' number can be eliminated.

2.1. Networks enlargement by new edges addition

Let be the initial network ant its graph G_0 histogram:

$$G_0\begin{bmatrix} k \\ N_k \end{bmatrix} = \begin{bmatrix} 1 & 2 & \dots & i & i+1 & \dots & j & j+1 & \dots & n \\ N_1 & N_2 & \dots & N_i & N_{i+1} & \dots & N_j & N_{j+1} & \dots & N_1 \end{bmatrix} \quad (9)$$

An elementary enlargement is made adding a single new edge, denoted by the operation:

$$G_{r+1} = e_{ij}(G_r), 1 \le i \le j \le n \quad (10)$$

where:

- G_r, G_{r+1} are two successive graphs;
- e_{ij} - connection of one nodes with *i* edges with the other one, with *j* edges.

The new graph's histogram will be:

$$G_{r+1} = \begin{cases} [1 \dots\dots i.\ \ i+1 \dots\dots j\ \ j+1 \dots n \\ N_1 \dots N_i - 1\ \ N_{i+1}+1 \dots N_j\ -1\, N_{j+1}+1 \dots N_n] \\ \qquad if\ j > i+1 \\ [1 \dots\dots i\ \ i+1\ \ i+2 \dots\dots n \\ N_1 \dots N_i - 1\ \ N_{i+1}\ \ N_{i+2}+1 \dots N_n] \\ \qquad if\ j = i+1 \\ [1 \dots\dots i\ \ i+1 \dots n \\ N_1 \dots N_i - 2\ \ N_{i+1}+2\ \dots N_n] \\ \qquad if\ j = i \end{cases} \quad (11)$$

Because we'll use these forms to compute $\langle k \rangle$ and $\langle k^2 \rangle$, the expressions (Eq.11) can be rewrite evidencing only the columns supporting modifications between two consecutive graphs G_r and G_{r+1} :

$$G_{r+1} = \begin{cases} \left[[A_1]\begin{matrix} i & i+1 & j & j+1 \\ N_i - 1 & N_{i+1}+1 & N_j - 1 & N_{i+1}+1 \end{matrix}\right] & if\ j > i+1 \\ \left[[A_2]\begin{matrix} i & i+1 & i+2 \\ N_i - 1 & N_{i+1} & N_{i+2}+1 \end{matrix}\right] & if\ j = i+1 \\ \left[[A_3]\begin{matrix} i & i+1 \\ N_i - 2 & N_{i+1}+2 \end{matrix}\right] & if\ j = i \end{cases} \quad (12)$$

where $A_{1,2,3}$ no changes from G_r to G_{r+1} .

In the same manner we perform graphs' modification eliminating a single edge. We'll symbolize this action adding to the elementary enlargement the minus sign: so, according with (Eq.11), $-e_{ij}$ represents the removal of connection between a node with *i* edges and an other one, with *j* edges. Of course this operation can be made only if there is a connection between the selected nodes.

In [2] we proved the next assertion: Let be G_o, G_F the initial, respectively the enlarged (final) graphs. Then the final graph can by obtained by ***m*** elementary enlargements, where:

$$\max(m) = \sum\left(N_k^F - N_k^0\right)_{N_k^F - N_k^0 > 0} = \sum\left(N_k^F - N_k^0\right)_{N_k^F - N_k^0 < 0} \quad (13)$$

2.2. Elementary enlargements' properties

Let attach to G_r and G_{r+1} their critical fractions $f_{C,r}$ and $f_{C,r+1}$.

We'll accept as obvious the assertion:

$$f_C^{\,r+1} \geq f_C^{\,r} \qquad (14)$$

On the basis of the elementary enlargement definition, using (Eq.10) and (Eq.12), the next properties of two successive elementary enlargements can be enounced:

I. Assumption:

For $i = 1 > n-1$,

$$e_{i,i+1}(G_r) \geq, e_{i,i+1}(G_r) \qquad (15)$$

Proof:
Let be:

$$x = \frac{<k^2>}{<k>} \qquad (16)$$

Thus,

$$df_c = \frac{1}{(1-x)^2}dx \qquad (17)$$

a) For $G_{r+1} = e_{ii}(G_r), i = 1:n-1$, using (12), we'll have consecutively:

$$<k_{r+1}^{\;2}> - <k_r^{\;2}> = \frac{4i+2}{\sum N} \qquad (18)$$

And

$$<k_{r+1}> - <k_r> = \frac{2}{\sum N} \qquad (19)$$

Accept that x' variance is due especially to $<k^2>$.We can do that for all $i \geq 2$.
Thus, with

$$df^{ii}{}_c = \frac{1}{(1-x)^2} \cdot \frac{4i+2}{\sum N} \qquad (20)$$

b) For $G_{r+1} = e_{i,i+1}(G_r), i = 1:n-2$,

$$<k_{r+1}^{\;2}> - <k_r^{\;2}> = \frac{2i+4}{\sum N} \qquad (21)$$

And

$$<k_{r+1}> - <k_r> = \frac{2}{\sum N} \qquad (22)$$

Accept that x' variance is due especially to $<k^2>$.We can do that for $i \geq 2$.

$$df^{i,i+1}{}_c = \frac{1}{(1-x)^2} \cdot \frac{4i+2}{\sum N} \qquad (23)$$

Comparing (20) and (23),

$$df_C^{i,i+1} > df_C^{i,i}$$

Hence, induces in the elementary enlargement's set one order relationship:

$$e_{11} \leq e_{12} \leq e_{22} \leq ... \leq e_{i-1,i} \leq ... \leq e_{n-1,n-1}$$

II. Let be $C = \{a,b,c,d\} \in N$, $k_{\min} \leq a \leq b \leq c \leq d < k_{\max}$ and $x,y,z,t \in C$

Hence, we can write a number of equalities between the couples of elementary enlargements,

$$e_{xy} + e_{zt} = e_{xz} + e_{yt} \qquad (24)$$

III. According to the convention for the signs associated to the elementary enlargements, in (Eq.24) we can move the terms following usual algebraic rules.

3. APPLICATIONS

3.1. Let be the initial graph, with 25 nodes and 28 edges, described by its histogram:

$$G_0 = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 19 & 5 & 0 & 1 \end{bmatrix}$$

The node numbers of the nearest scale free G'$_F$ graph, having the slope $tg(S) > 2$ and keeping as constant the nodes' number $\sum N(k) = 25$ found by a linear regression method are: N'$_2$=14.0238 N'$_3$= N25.8551 N'$_4$=3.1506 N'$_5$=1.9482. Rounding these values, we obtain the final graph's degree distribution as:

$$G_F = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 14 & 6 & 3 & 2 \end{bmatrix}$$

and:

$$\begin{pmatrix} \text{k} \\ \text{P(k)} \end{pmatrix} = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0.56 & 0.24 & 0.12 & 0.08 \end{bmatrix}$$

Observe that the edges number is 34.

That means we can extend the original graph by adding 6 newedges.

One possible set E of elementary extensions from G_o to G_F can be:

$$E_1 = \{e_{22}, e_{23}, e_{23}, e_{33}, e_{24}\}$$

Between the couples of elementary enlargements, from (Eq.16), we have successively:

(1) $e_{22} + e_{33} = e_{23} + e_{23}$

(2) $e_{22} + e_{34} = e_{23} + e_{24}$

(3) $e_{22} + e_{44} = e_{24} + e_{24}$

(4) $e_{23} + e_{34} = e_{24} + e_{33}$

(5) $e_{23} + e_{44} = e_{24} + e_{33}$

(6) $e_{24} + e_{34} = e_{23} + e_{44}$

(7) $2e_{24} + e_{34} = e_{23} + e_{44}$

Hence, using these equalities, one other solution is:

$$E_2 = \{e_{34}, e_{33}, e_{23}, e_{22}, e_{22}\}$$

The initial and the final graphs are depicted in fig.1: the added connections are evidenced.

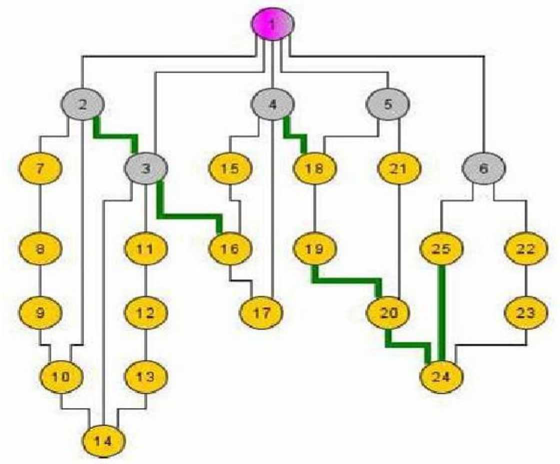

Fig.1. The initial and the new networks (E_2)

Performing the elementary extensions of E_1 in an indefinite order, a G_1, G_1,…, G_5 graph array is generated. We'll compute with (Eq.4), for all $G_i, i = 1:5$, the critical fraction f_c : The results are contained in Table 1.

Table 1

Graph	**e**					**fC**	**ΔfC**
	k	**2**	**3**	**4**	**5**		
G_0	e_{22}	19	5	0	1	0.3409	
G_1	e_{23}	17	7	0	1	0.3750	0.0241
G_2	e_{23}	16	7	1	1	0.4151	0.0401
G_3	e_{33}	15	7	2	1	0.4483	0.0332
G_4	e_{24}	15	5	4	1	0.4844	0.0361
G_5		**14**	**6**	**3**	**2**	0.5143	0.0299

Let perform the successive enlargements from E_2, beginning with the highest indices :

Table 2

Graph	**e_{ij}**					**f_C**	**Δf_C**
	k	**2**	**3**	**4**	**5**		
G_0		19	5	0	1	0.3409	
G_1	e_{34}	19	4	0	2	0.4148	**0.0739**
G_2	e_{33}	19	2	2	2	0.4561	**0.0413**
G_3	e_{23}	18	2	3	2	0.4839	**0.0278**
G_4	e_{22}	16	4	3	2	0.5000	0.0161
G_5	e_{22}	**14**	**6**	**3**	**2**	0.5143	0.0163

Observe that, applying the E_2 enlargement strategy, at the third step we attain a value of the critical fraction critical achieved for E_1 only in the fourth step.

3.2. Let study the initial graph described by its histogram

$$G_0 = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 16 & 3 & 1 & 5 \end{bmatrix}$$

Alike in the last example, the most nearer scale –free graph is also:

$$G_F = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 14 & 6 & 3 & 2 \end{bmatrix}$$

Thus, G_0 have three high connected nodes in excess. Theoretically we solve the problem eliminating these nodes, implying the deletion of connected edges (tab3) :

Table 3

Graph	Elementary enlargement **k**	N_k				f_c
		2	**3**	**4**	**5**	
G_0		16	3	1	5	0.5679
G_1	- e_{45}	16	4	1	4	0.5405
G_2	- e_{44}	16	4	3	2	0.5000
G_3	e_{22}	**14**	**6**	**3**	**2**	0.5143

The initial and the final graphs are depicted in fig. 2: the dropped lines represent the deleted connections. Actually, we needn't to physically eliminate them. The example is choosing to illustrate that the initial network has 35 edges, but to obtain a robust network, even 35 edges are enough.

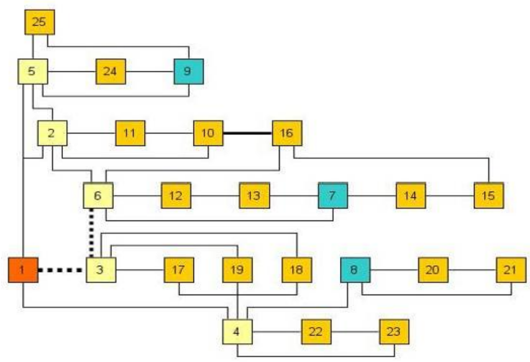

Fig.2. The initial and the new networks

In this case the result seams to be ambiguous. In fact is very possibly to work with over connected power grids. The scale –free solution is the most economic way to assure a desired critical fraction with a minimal edges number.

CONCLUSIONS

The ideas presented in our paper offers useful tools to extend step by step one vulnerable network to the nearest robust network. Measuring the critical fraction's variance is hence an appropriate pointer helping to choose optimal enlargement strategies, consisting in taking decisions on the elementary enlargements' order. Analyzing the applications' result, we can study the power grids', robustness oriented enlargement with the proposed tools.
As the examples illustrates, the realistic level of the grids' robustness can be achieves also by renouncing to made all, mathematically necessary enlargement steps. Also, these tools give the departing point for the economically acceptable solutions, depending to the real geographic locations o the power grid's nodes.

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