

# POWER FLOW TRACING METHOD FOR ELECTRICITY TRANSMISSION AND WHEELING PRICING

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**Abstract:** The power transmission costs, which are charged to the market participants, are a central issue of the new deregulated electricity markets. The paper presents a methodology for transmission cost allocation based on the principle of proportional sharing states that any power flow leaving a bus is made up of the flows entering that bus in a proportional manner, thus satisfying Kirchhoff first law. A properly established cost provides economical signals for short-term and long-term recovery current expenses and for a fair cost allocation to market participants. An original method of transmission cost allocation considering the power losses is proposed. The appropriate software tool has been developed in Mathematica environment. The results are presented and analyzed for the Test 25 test power system.

**Keywords:** transmission system, tracing method, cost allocation, proportional sharing principle, power flow

## 1. INTRODUCTION

Electricity transmission and wheeling service pricing becomes a more complex and more important task within the ongoing deregulation of electric power industry. It has significant impacts on the market efficiency, the development of transmission systems, the siting of power plant, the demand growth and its geographical distribution. Transmission pricing discusses how to allocate the entire cost of a transmission system among all the system users [1], [2], [3]. Because monetary flows are related to the nodal prices, the impact of generators and loads on operation constraints and the interactive correlation between active and reactive power can be considered.

Total transmission service cost is separated into more practical line-related costs and system wide cost, and can be flexibly distributed between generators and loads. It is necessary to find out the contributions of each generator and load to transmission system power flows

before a reasonable distribution of line-related costs and system-wide cost can be obtained [4], [5]. This paper uses the Kirschen tracing method to calculate the contributions of generators and loads to power flows and active losses [6], [7]. This method is based on the proportional sharing principle. The novelty of the proposed approach refers to AC power flow based version.

The buses and the network elements are organized in homogeneous groups based on the following concepts: the domain of the generator, the commons and the links [8]. The domain of a generator represents a subset of buses, which are supplied by certain generator. The commons of a generator are defined as a subset of neighboring buses supplied by the same generators. Having the buses divided into commons, each network element can be either internal to a common (for example, it connects two buses which are part of the same common) or external (for example, it connects two buses belongs to different commons). A link is represented by one or more external branches connecting the same commons.

In order to illustrate this method, the 25-bus test power system is used, designed within the Power Systems Department, from "Politehnica" University of Timisoara, Romania [12].

Concerning the structure of the paper, following the Introduction presented within the 1<sup>st</sup> section, the 2<sup>nd</sup> one is focusing on describing the method. The results are presented and discussed within the 3<sup>rd</sup> section. The case study power system is presented within the same section. The last section presents the conclusion of the paper.

## 2. METHOD DESCRIPTION

Kirschen method organizes the network's buses and branches in homogeneous groups according to the following concepts: the domain of generator, the commons and the links [9]-[10]. These concepts are used to obtain the state graph of generators contribution to consumers in a domain and for determining the generator contributions to individual consumers and power flows.

The method is applied independently both for active and reactive power, based on complete AC power flow [11].

The domain of a generator represents a subset  $N$  of  $n$  buses, which are supplied by this generator. A particular bus belongs to a generator domain if there is a “trace” through the transmission network for which the direction of power flow is from the generator to the bus. In the manner of domain definition results that there is an overlap between the domains of the various generators. It is noted that the “active domain” of a generators does not usually cover the same set of buses as its “reactive domain”.

The generator common is a subset of “neighbored” buses from the subset  $N$ , which are supplied by the same generators. The subsets of buses which are not connected and not supplied by the same generators are treated as separate commons. A bus therefore belongs to only one common. The rank of a common is defined as the number of generators supplying the buses within this common.

Once the buses have been divided into commons, each network element within the common connects two buses which belong to the same common. If a network branch is inside of a common, then it will connect two buses which are belonging to different commons. A link consists by one or more external network branches connecting the same commons. It is very important to note that the actual flows in all branches of a link are all in the same direction. Furthermore, the flow direction in a link is always from a common of rank  $i$  to a common of rank  $j$ , where  $j$  is always greater than  $i$ .

Knowing the direction of the power flow through the network elements and being defined the commons and the links, the graph theory can be applied. The state graph for the problem studied can be obtained using the following representation conventions: the graph vertices correspond to commons, graph arcs correspond to links between commons and the arc orientation is given by the power flows. The initial buses of arcs correspond to commons of lower rank, while the terminal buses correspond to commons of higher rank.

The state graph provides only a qualitative view of power system. The inflow of a common is defined as the sum of the injected power by the sources connected to buses located in this common and the power imported in this common from other common by links. This inflow is always positive.

The calculus of generator contribution to various commons ( $D$  is the set of commons) is made in the following manner [11]:

$$P_{jk}^i = w_j^i \cdot P_{jk} \quad (1)$$

$$P_k = \sum_{j \in D} P_{jk} \quad , \quad k \in D \setminus j \quad (2)$$

$$w_k^i = \frac{\sum_{j \in D} P_{jk}^i}{P_k} \quad , \quad k \in D \quad (3)$$

where:  $w_j^i$  – the contribution of the  $i$  generator to the load and the outflow of the  $j$  common;  $w_k^i$  – the contribution of the  $i$  generator to the load and the outflow of the  $k$  common;  $P_{jk}$  – the flow on the link between the  $j$  and  $k$  commons;  $P_{jk}^i$  – the flow on the link between the  $j$  and  $k$  commons due to the  $i$  generator;  $P_k$  – the inflow of the  $k$  common.

The inflow of the root buses of state graph is produced entirely by the generators embedded in these commons. The proportion of the outflow traceable to each of these generators can therefore be readily computed and propagated to commons of higher rank.

Knowing the common a bus belongs to and the contributions of each generator to each common, therefore gives the ability to compute how much power each generator contributes to each load. It also makes it possible to compute what proportion of the use of each branch can be apportioned to each generator. For network elements linking buses in separate commons, the proportion of usage should be based on the contribution of the generator to the lower ranked common.

Within the methodology the hypothesis regarding the contribution of the generators to the power losses on a branch in proportion to the rate of use of this branch is adopted. So the power losses allocation can be performed.

### 3. CASE STUDY

A 25 buses test power system has been used for the analyses [12]. It has 29 branches. It was developed based on the south-west side of the Romanian Power System. 6 P-U buses (the slack bus is bus number 1) and 19 P-Q buses, the voltage level for 2 buses is 400 kV, 8 buses are at 220 kV, 10 buses at 110 kV, one bus at 24 kV, 2 buses at 15 kV and 2 buses at 10 kV. In this particular operating condition, 4 P-Q buses and 3 P-U buses have zero consume power and the source from bus 6 is a synchronous compensator (Fig. 1).

17 network elements are electrical overhead lines (one of 400 kV, 8 of 220 kV and 8 of 110 kV) and one is under-ground cable; 5 are transformers and 6 auto-transformers. The generated and consumed active power, for the 25 buses test system is synthesized in Table 1. The active power flow through the network elements is presented in Table 2.

Table 1. Configuration of the P-U and P-Q buses

Bus	U [kV]	P <sub>C</sub> [MW]	P <sub>g</sub> [MW]	Bus	U [kV]	P <sub>C</sub> [MW]	P <sub>g</sub> [MW]
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Bus	U [kV]	P <sub>C</sub> [MW]	P <sub>g</sub> [MW]	Bus	U [kV]	P <sub>C</sub> [MW]	P <sub>g</sub> [MW]
1	24	80	738.75	14	221.854	237	0
2	15.8	8	1042.68	15	224.237	0	0
3	15.5	80	680.68	16	223.315	0	0
4	10.6	0	50	17	118.353	0	0
5	114.334	0	20	18	118.535	120	0
6	10.815	0	4	19	116.18	32	0
7	405.053	350	0	20	117.199	22	0
8	400.23	530	0	21	113.287	20	0
9	236.179	156	0	22	113.52	35	0
10	236.791	175	0	23	113.538	12	0
11	238.393	400	0	23	113.197	58	0
12	235.084	0	0	25	112.132	24	0
13	222.341	170	0				

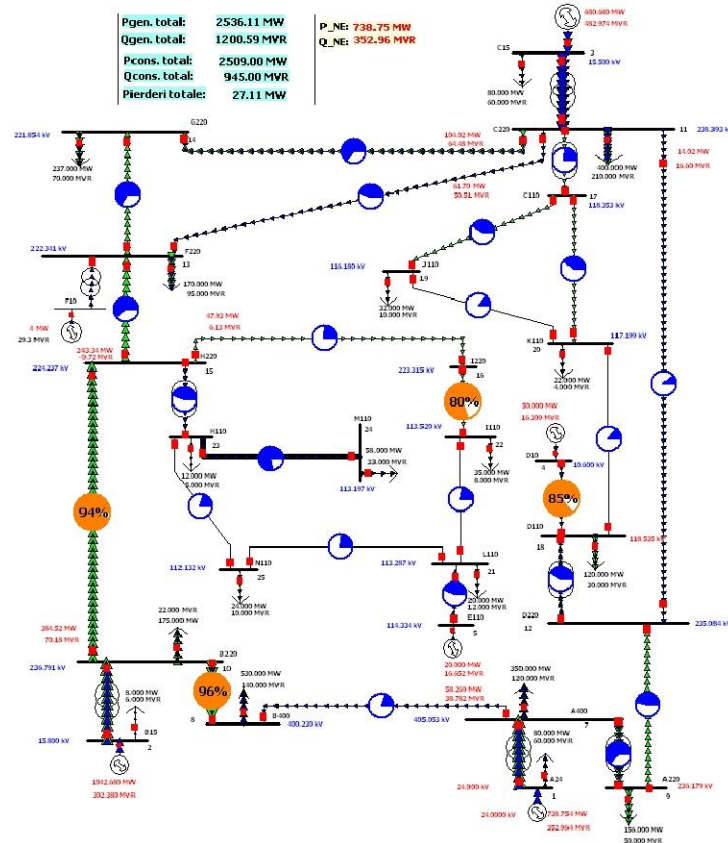


Fig. 1. The case study power system

Table 2. Active power flows on the system branches

Bus i	Bus j	P <sub>ij</sub> [MW]	ΔP [MW]	L <sub>ij</sub> [km]	Bus i	Bus j	P <sub>ij</sub> [MW]	ΔP [MW]	L <sub>ij</sub> [km]
1	7	657.5	1.26	153	6	13	3.9	0.06	42
7	9	249.2	0.39	173	11	14	102.6	1.38	3
2	10	1032.1	2.61	51	13	14	134.4	0.63	122
10	8	471.8	0.7	31	23	25	11.5	0.09	72
7	8	58.1	0.12	29	23	24	58.1	0.12	32
17	19	23.8	0.22	94	10	15	373	11.51	15
17	20	23	0.22	72	15	23	81.7	0.11	72
3	11	598.6	2.1	68	15	16	47.9	0.1	54
12	11	14	0.09	86	22	21	12.8	0.03	86
11	17	46.9	0.03	72	16	22	47.8	0.03	12
4	18	49.8	0.24	136	20	19	8.4	0.05	37
9	12	92.1	0.73	38	18	20	7.6	0.03	42
12	18	78	0.08	153	5	21	19.9	0.11	3
11	13	60.5	1.22	173	21	25	12.6	0.08	122
15	13	240.6	2.77	51					

The active power losses for the entire power system are 27.11 MW.

This paper proposes a simple method of transmission cost allocation related to power losses, based

on results obtained with Kirschen method for operating condition without losses. A software tool in Mathematica environment has been developed.

The system contains 6 generators, so will result 6 commons (Fig. 2). The common 5, corresponding to source of bus 5, will contain the largest number of buses, even the generated power is 20 MW and the consumed power is 149 MW. The generator of common 3 produces 680.85 MW

for a load of 534 MW. It is clear that this generator will supply other areas through the link 3-6 (the lines 11-13 and 11-14), for a total amount of 163.1 MW. Also, commons 3 and 4 are linked by the 4-3 line, with the normal sense of the power flow from bus 4 to bus 3. This link consists of lines 12-11 and 18-20, the active power transfer being 21.6 MW.

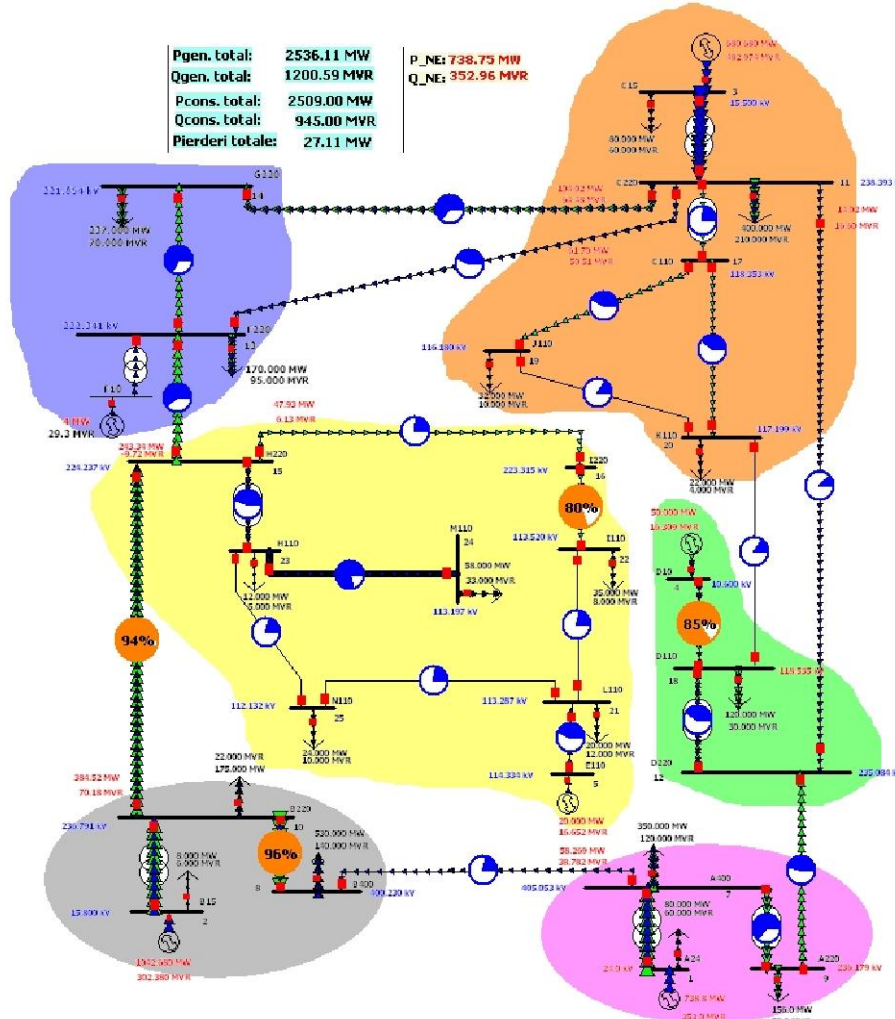


Fig. 2. The choice of commons

Table. 3. Defining the commons in the system

Common	Buses	P <sub>generated</sub> [MW]	P <sub>consumed</sub> [MW]
1	1, 7, 9	738.75	586
2	2, 8, 10	1042.68	713
3	3, 11, 19, 20	680.68	534
4	4, 18, 12	50	120
5	5, 15, 16, 21, 22, 23, 24, 25	20	149
6	6, 14, 13	4	407

Table. 4. Defining the links between the commons

Common i	Common j	Network elements	P <sub>transfer</sub> [MW]
1	2	7-8	58.1
2	5	10-15	373
5	6	15-13	240.6
1	4	9-12	92.1
4	3	12-11, 18-20	21.6
3	6	11-13, 11-14	163.1

The graph obtained is presented in Fig. 3. The root bus is selected to be the slack bus 1. Beginning from this bus, his

participation on the links 1-4 and 1-2 is obvious. The source 1 will participate on link 4-3 with 6.51 MW; source 4 with

15.08 MW. Through the link 2-5 the contributions are: 19.69 MW (generator of bus 1) and 353.31 MW (generator of bus 2). The computing algorithm ends with common 6. To meet the load power in common 6, all the system sources are going to participate as follows:

- generator 1 - 13.57 MW, generator 2 - 216.3 MW;

- generator 3 - 158.08 MW; generator 4 - 3.5 MW;
- generator 5 - 12.24 MW; generator 6 - 4 MW.

The generator contribution to power flow through the network elements is computed (Table 5).

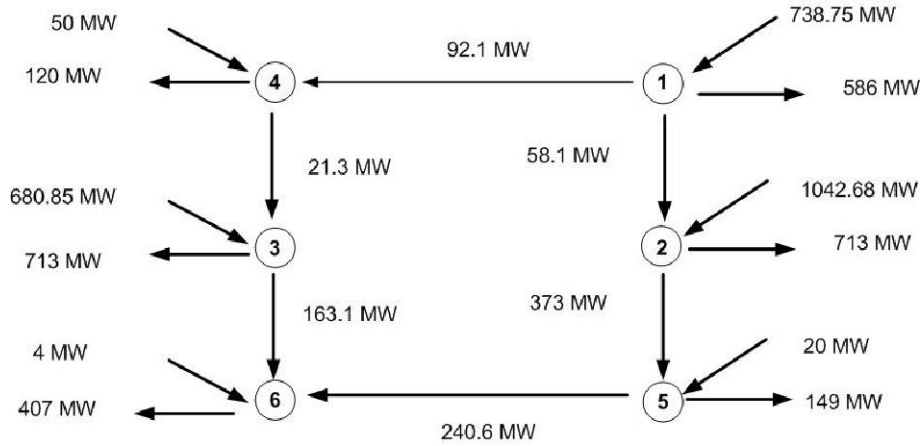


Fig. 3. The state graph

Table. 5. The contribution of the generators to power flow through the network elements (the operating condition considering the active power losses)

Bus i	Bus j	$P_{ij}$ [MW]	$P_{ij}^{g1}$ [MW]	$P_{ij}^{g2}$ [MW]	$P_{ij}^{g3}$ [MW]	$P_{ij}^{g4}$ [MW]	$P_{ij}^{g5}$ [MW]	$P_{ij}^{g6}$ [MW]
1	7	657.5	657.5	0	0	0	0	0
7	9	249.2	249.2	0	0	0	0	0
2	10	1032.1	0	1032.1	0	0	0	0
10	8	471.8	0	471.8	0	0	0	0
7	8	58.1	58.1	0	0	0	0	0
17	19	23.8	23.3	0	0.5	0	0	0
17	20	23	22.5	0	0.5	0	0	0
3	11	598.6	0	0	598.6	0	0	0
12	11	14	14.0	0	0	0	0	0
11	17	46.9	45.8	0	1.1	0	0	0
4	18	49.8	0	0	0	49.8	0	0
9	12	92.1	92.1	0	0	0	0	0
12	18	78	78.0	0	0	0	0	0
11	13	60.5	59.1	0	1.4	0	0	0
15	13	240.6	0	240.6	0	0	0	0
6	13	3.9	0	0	0	0	0	3.9
11	14	102.6	100.3	0	2.3	0	0	0.0
13	14	134.4	39.1	0	93.8	0	0	1.5
23	25	11.5	0	11.5	0	0	0	0
23	24	58.1	0	58.1	0	0	0	0
10	15	373	0	373.0	0	0	0	0
15	23	81.7	0	81.7	0	0	0	0
15	16	47.9	0	47.9	0	0	0	0
22	21	12.8	0	12.8	0	0	0	0
16	22	47.8	0	47.8	0	0	0	0
20	19	8.4	7.5	0	0.1	0.8	0	0
18	20	7.6	4.6	0	0	3.0	0	0
5	21	19.9	0	0	0	0	19.9	0
21	25	12.6	0.1	4.9	0	0	7.6	0

Finally, the transmission cost allocated to generators is computed, taking into account the active power losses. The unit transmission cost on lines of 2 \$/MW·km is considered. The results are synthesized in Table 6.

**Table 6. The allocation of transmission costs to the generators (including the active power losses)**

Bus i	Bus j	Line cost ij [\$]	P <sub>ij</sub> [MW]	The allocation of transmission costs					
				Gen. 1 [\$]	Gen. 2 [\$]	Gen. 3 [\$]	Gen. 4 [\$]	Gen. 5 [\$]	Gen. 6 [\$]
1	7	360	657.5	236700	0	0	0	0	0
7	9	612	249.2	152510.4	0	0	0	0	0
2	10	24	1032.1	0	24770.4	0	0	0	0
10	8	306	471.8	0	144370.8	0	0	0	0
7	8	346	58.1	20102.6	0	0	0	0	0
17	19	102	23.8	2376.6	0	51	0	0	0
17	20	62	23	1395	0	31	0	0	0
3	11	58	598.6	0	0	34718.8	0	0	0
12	11	188	14	2632	0	0	0	0	0
11	17	144	46.9	6595.2	0	158.4	0	0	0
4	18	136	49.8	0	0	0	6772.8	0	0
9	12	172	92.1	15841.2	0	0	0	0	0
12	18	144	78	11232	0	0	0	0	0
11	13	272	60.5	16075.2	0	380.8	0	0	0
15	13	76	240.6	0	18285.6	0	0	0	0
6	13	550	3.9	0	0	0	0	0	2145
11	14	232	102.6	23269.6	0	533.6	0	0	0
13	14	58	134.4	2267.8	0	5440.4	0	0	87
23	25	84	11.5	0	966	0	0	0	0
23	24	6	58.1	0	348.6	0	0	0	0
10	15	244	373	0	91012	0	0	0	0
15	23	144	81.7	0	11764.8	0	0	0	0
15	16	64	47.9	0	3065.6	0	0	0	0
22	21	30	12.8	0	384	0	0	0	0
16	22	144	47.8	0	6883.2	0	0	0	0
20	19	108	8.4	810	0	10.8	86.4	0	0
18	20	172	7.6	791.2	0	0	516	0	0
5	21	24	19.9	0	0	0	0	477.6	0
21	25	74	12.6	7.4	362.6	0	0	562.4	0
Total [\$]				492606.2	302213.6	41324.8	7375.2	1040	2232
Total transmission cost [\$]				846791.8					

In case of consumers, working in a similar manner, the final results presented in Table 7 are obtained (transmission costs allocated to consumers in the presence of active power losses).

*Table 7. The transmission cost allocation to consumers (including the active power losses)*

Bus i	Bus j	P <sub>ij</sub> [MW]	The allocation of transmission costs							
			Load 7 [\$]	Load 8 [\$]	Load 9 [\$]	Load 10 [\$]	Load 11 [\$]	Load 13 [\$]	Load 14 [\$]	Load 18 [\$]
1	7	657.5	126000	20916	56160	0	3290.9	277.4	441.5	26366.2
7	9	249.2	0	0	95472	0	5594.4	471.8	750.6	44822.5
2	10	1032.1	0	11500.5	0	4265.8	0	3268.9	2584.3	0
10	8	471.8	0	144370.8	0	0	0	0	0	0
7	8	58.1	0	20102.6	0	0	0	0	0	0
17	19	23.8	0	0	0	0	0	0	0	0
17	20	23	0	0	0	0	0	0	0	0
3	11	598.6	0	0	0	0	22669.8	0	7325.7	0
12	11	14	0	0	0	0	1718.5	144.9	230.6	0
11	17	46.9	0	0	0	0	0	0	0	0
4	18	49.8	0	0	0	0	0	0	0	6359.4
9	12	92.1	0	0	0	0	1572.3	132.6	210.9	12597.2
12	18	78	0	0	0	0	0	0	0	10546.5
11	13	60.5	0	0	0	0	0	9190.3	7265.7	0
15	13	240.6	0	0	0	0	0	10212	8073.6	0

Bus i	Bus j	P <sub>ij</sub> [MW]	The allocation of transmission costs							
			Load 7 [\$]	Load 8 [\$]	Load 9 [\$]	Load 10 [\$]	Load 11 [\$]	Load 13 [\$]	Load 14 [\$]	Load 18 [\$]
6	13	3.9	0	0	0	0	0	1197.9	947.1	0
11	14	102.6	0	0	0	0	0	0	23803.2	0
13	14	134.4	0	0	0	0	0	0	7795.2	0
23	25	11.5	0	0	0	0	0	0	0	0
23	24	58.1	0	0	0	0	0	0	0	0
10	15	373	0	0	0	0	0	33041.6	26122.3	0
15	23	81.7	0	0	0	0	0	0	0	0
15	16	47.9	0	0	0	0	0	0	0	0
22	21	12.8	0	0	0	0	0	0	0	0
16	22	47.8	0	0	0	0	0	0	0	0
20	19	8.4	0	0	0	0	0	0	0	0
18	20	7.6	0	0	0	0	0	0	0	0
5	21	19.9	0	0	0	0	0	0	0	0
21	25	12.6	0	0	0	0	0	0	0	0
Total [\$]			126000	196889.9	151632	4265.8	34845.9	57937.4	85550.7	100691.8

Bus i	Bus j	P <sub>ij</sub> [MW]	The allocation of transmission costs							
			Load 19 [\$]	Load 20 [\$]	Load 21 [\$]	Load 22 [\$]	Load 23 [\$]	Load 24 [\$]	Load 25 [\$]	
1	7	657.5	2047.4	1200.6	0	0	0	0	0	0
7	9	249.2	3358.2	2040.9	0	0	0	0	0	0
2	10	1032.1	0	0	190.8	853.2	292.5	1413.8	400.6	0
10	8	471.8	0	0	0	0	0	0	0	0
7	8	58.1	0	0	0	0	0	0	0	0
17	19	23.8	2427.6	0	0	0	0	0	0	0
17	20	23	62	1364	0	0	0	0	0	0
3	11	598.6	3419.8	1303.5	0	0	0	0	0	0
12	11	14	439.2	98.8	0	0	0	0	0	0
11	17	46.9	3585.6	3168	0	0	0	0	0	0
4	18	49.8	123.8	289.6	0	0	0	0	0	0
9	12	92.1	754.6	573.6	0	0	0	0	0	0
12	18	78	205.3	480.2	0	0	0	0	0	0
11	13	60.5	0	0	0	0	0	0	0	0
15	13	240.6	0	0	0	0	0	0	0	0
6	13	3.9	0	0	0	0	0	0	0	0
11	14	102.6	0	0	0	0	0	0	0	0
13	14	134.4	0	0	0	0	0	0	0	0
23	25	11.5	0	0	0	0	0	0	0	966
23	24	58.1	0	0	0	0	0	348.6	0	0
10	15	373	0	0	1928.9	8623.5	2956.6	14290.4	4048.6	0
15	23	81.7	0	0	0	0	1732.2	8372.5	1660.1	0
15	16	47.9	0	0	502.5	2246.5	0	0	316.6	0
22	21	12.8	0	0	235.6	0	0	0	148.4	0
16	22	47.8	0	0	1128.2	5044.2	0	0	710.8	0
20	19	8.4	907.2	0	0	0	0	0	0	0
18	20	7.6	733.6	573.6	0	0	0	0	0	0
5	21	19.9	0	0	293	0	0	0	0	184.6
21	25	12.6	0	0	0	0	0	0	0	932.4
Total [\$]			18064.3	11092.8	4279	16767.4	4981.4	24425.3	9368.1	0
Total transmission cost [\$]			846791.8							

#### 4. Conclusion

The authors are proposing a new method of assessing the transmission cost related to active power losses. The Kirschen method is suitable to estimate the performance of the system. This method is simple, intuitive and is based on complete AC power flow. The cost

allocation has been performed separately for sources and consumers, tracing the active power flow. The two components of the transmission cost allocation can be weighted differently (in the range 0÷1, their sum is obviously 1). Due to obvious influence of the active power losses, it is recommended not to be neglected.

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