THE EVALUATION OF THE DURABILITY OF A BATCH OF ELECTRICAL ENGINE STARTERS BASED ON A WEIBULL MODEL

CSUZI. I.*, CÂRLAN M.**, COSTEA M.***
*S.C. Oradea Transport Local S.A , Oradea
**S.C. Formenerg S.A, Bucharest
*** Lumina -The University of South- East Europe
marius.costea@lumina.org

Abstract - This paper presents a method to evaluate the durability of a batch of electrical starters , fitted on buses used by the public transportation company of the municipality of Oradea. These starters have a short transient working regime, characterized by frequent start-stop usage, especially during the cold season. Based on statistical sample obtained from the exploitation documentation, it was proven that the temporal evolution of these starters is characterized by a Weibull model, but with a conspicuos tendincy towards the Rayleigh version – a distinctive model of equipments that have an operational regime characterized by strong attrition. Also, this paper analyzes the topic of acquiring this sort of equipments based on preliminary testing. These tests are meant to assess the probable durability level of these operational entities.

Keywords: ndard deviation, variation coefficient, empirical function, calculation quantile, test quantile

1. INTRODUCTION

The temporal evolution of elements belonging to a statistical sample, extracted from the exploitation records of those entities, may be of a certain nature in what concerns the operational times distribution. But in most cases, the distribution model may be exponential, or, more frequently, if the equipment in question registered a certain degree of wear and tear, the model concerning the operational behaviour over time, is Weibull type.

A first phase in solving such a problem consists in validating one of these two hypotheses. The calculations imposed by the testing of the operational times distribution will be carried out in accordance with the protocol imposed by each model.

2. CASE STUDIES

2.1. Case study 1

Table 1 shows a statistics regarding the temporal evolution of failures (fault conditions) of engine starter in the equipping of some local transport buses. In this table, the meanings of the values included are:

- $[t_i^{min}, t_i^{max}]$ represents the time interval when the failures (faults) occurred;
- r_i is the number of functioning stops during that interval;
- $\bar{t}$ is average of failure times.

Based on these elements, the validation calculation of these two hypotheses will be performed: exponential distribution or Weibull.

Table 1. The temporal evolution of failures

i	1	2	3	4	5	6	7	8	9
Interval - hours	0–1000	1001-2000	2001-3000	3001-4000	4001-5000	5001-6000	6001-7000	7001-8000	8001-9000
r_i	15	4	3	0	7	7	9	5	2

2.1.1. The exponential model. Hann-Shaphiro-Wilk test [1], [2], [4], [5]

Based on the data showed in columns 5 and 9 of table 2, W_0 calculation quantile is determined:

$$W_0 = \frac{\sum_i (t_i^{max} - \bar{t})\, r_i}{(\sum t_i^{max} \cdot r_i)^2}. \qquad (1)$$

Results $W_0 = 0{,}0076$.

The exponential hypothesis is valid only if the following inequality is observed:

$$W_{n;p}^{inf} < W_0 < W_{n;p}^{sup}, \qquad (2)$$

where n represents the statistical sample's volume, P – plausibility threshold (complementary value to α risk factor). Usually, P=0.95, seldom P=0.90.

According to the test's quantiles, one can notice that inequality (2) is not fulfilled:

$$W_{n=9;p=0{,}95}^{inf} = 0{,}025 > W_0 = 0{,}0076 < W_{n=9;p=0{,}95}^{sup} = 0{,}205.$$

Therefore, the statistical sample does not validate the exponential model.

Table 2. Necessary calculation data for ascertainment of exponential nature of fault conditions distribution

i	$t_i^{\min};t_i^{\max}$	$t_i^{\max}$	r_i	$r_i \cdot t_i^{\max}$	$\bar{t}$	$t_i^{\max}-\bar{t}$	$(t_i^{\max}-\bar{t})^2$	$(t_i^{\max}-\bar{t})^2 r_i$
1	0000-1000	1000	15	15000	4423	-3423	117716929	175753935
2	1001-2000	2000	4	8000	4423	-2423	5870929	23483716
3	2001-3000	3000	3	9000	4423	-1423	202492	6074787
4	3001-4000	4000	0	0	4423	-423	178929	0
5	4001-5000	5000	7	35000	4423	577	332929	2330503
6	5001-6000	6000	7	42000	4423	1577	2486929	17408503
7	6001-7000	7000	9	63000	4423	2577	6640929	59768361
8	7001-8000	8000	5	40000	4423	3577	12794929	63974645
9	8001-9000	9000	2	18000	4423	4577	20948929	41897858
$\sum_i$	*	*	52	230000	*	*	62996361	390692308

2.1.2. Weibull biparametric model. Mann-Singpurwalla-Gupta test [3], [4], [7], [8]

According to this test, the distribution of failures (faults) is Weibull validated, if the inequality below is checked:

$$S_c < S_{n;p}, \tag{3}$$

where S_c represents the calculation quantile and $S_{n;p}$ is the test's quantile. Table 3 shows the necessary values to determine S_c calculation quantile; t_i, r_i have the meaning previously shown; $\Delta E(z_i)$ expresses Mann statistics,

l_i and l_j sizes are calculated as follows:

$$l_i = \frac{r_{i+1} \cdot \ln t_{i+1} - r_i \cdot \ln t_i}{\Delta E(z_i)}. \tag{4}$$

Size l_j is obtained as follows: j is determined,

$$j \ge [\frac{n}{2}]+1 \tag{5}$$

where $[\frac{n}{2}]$ expresses the integer part of the fraction:

$j \ge [\frac{9}{2}]+1 = [4,5]+1 = 4+1 = 5.$

Therefore j belongs to lines: $j \in \{5;6;7;8\}$.

The following calculation quantile results:

$$S_c = \frac{\sum_j l_j}{\sum_i l_i} \tag{6}$$

The calculation data sample is Weibull validated if the following inequality is valid:

$$S_c < S_{n;p}. \tag{7}$$

Indeed, Weibull validation condition is fulfilled:
$S_c = 0,6898 < S_{n=9;p=0,95} \in [0,72;0;95]$.

Calculation of Weibull distribution parameters

In the case of Weibull type of distribution, the method of moments proposed by Pearson [5] will be used to determine this distribution's parameters.
Based on the data in table 2, the following values result:

$$\bar{t} = \frac{\sum_i t_i \cdot r_i}{\sum_i r_i}, \tag{8}$$

$$\sigma = \sqrt{\frac{\sum_i (t_i - \bar{t})^2 \cdot r_i}{n}} \tag{9}$$

The following are obtained:
- average time (average time interval between two consecutive failures) :
$\bar{t} = 4423$ hours;
- squared average deviation (standard deviation):
$\sigma = 2741$ hours;
- variation coefficient,

$$CV = \frac{\sigma}{\bar{t}} = 0,62. \tag{10}$$

Depending on the variation coefficient, the following values are obtaining: $k = 1,65$, $g_k = 0,557$ and $V_k = 0,8944$, where:

$$V_k = \Gamma(\frac{1}{k}+1). \tag{11}$$

Weibull distribution parameter is deducted:

$$\theta = \frac{\sigma}{g_k}. \tag{12}$$

After substitution we obtain $\theta = 4921$ hours. In the end, the durability of the engine starters is obtained:

$$D(t) = \theta \cdot \Gamma(\frac{1}{k}+1) \tag{13}$$

After substitution we obtain $D(t) = 4401$ hours.

Table 3. Calculation data regarding validation of Weibull nature of fault conditions distribution

i	t_i	$\ln t_i$	r_i	$r_i \ln t_i$	$r_i \ln t_i$	$r_{i+1} \cdot \ln t_{i+1} - r_i \cdot \ln t_i$	$\Delta E(Z_i)$	l_i	l_j
1	1000	6.90776	15	103.61633	0.00000		1.060046	-	-
2	2000	7.60090	4	30.40361	18.21000	18.21000	0.566042	32.17076	-
3	3000	8.00637	3	24.01910	24.01910	5.80910	0.409457	14.18733	-
4	4000	8.29405	0	0.00000	30.40361	6.38451	0.337763	18.90234	-
5	5000	8.51719	7	59.62035	44.93600	14.53239	0.304777	47.68204	47.68204
6	6000	8.69951	7	60.89660	59.62075	14.68475	0.297949	49.28612	49.28612
7	7000	8.85367	9	79.68299	60.89660	1.27585	0.322189	3.95994	3.95994
8	8000	8.98720	5	44.93598	79.68300	18.78640	0.424958	44.20766	44.20766
9	9000	9.10498	2	18.20996	103.61693	23.93393		-	-
	*	*	52	*	*	*	*	210.39619	145.13576

Compared to the average time between two consecutive failures, $\bar{t}$, durability is approximately 5% more reduced:

$$q = [1 - \frac{D(\mathrm{t})}{\bar{t}}] \cdot 100 \,. \qquad (14)$$

After substitution we obtain $q = 0{,}5\%$.

A durability assessment prediction method

According to [4], [9], [10] the average durability can be estimated based on the relation:

$$\tilde{D}(\mathrm{t}) = \frac{1}{r}[\sum_j t_j + (\mathrm{n} - \mathrm{r})\,\mathrm{t}_r^*], \qquad (15)$$

where:

- $\mathrm{j} = \overline{1;r}$;
- $\tilde{D}(\mathrm{t})$ means anticipated (forecasted) durability;
- t_j is the operating time until the failure;
- t_r^* is the time reserved for testing;
- r is the number of failed entities;
- is n the number of tested sample entities.

2.2. Case study 2

Let' s consider a batch of n=10 electrical engine starters which is to be subjected to testing operation for a time interval of $t^* = 2500$ hours, during which three failures occurred (table 4):

Table 4. Necessary calculation elements for evaluation of electrical engine-starters durability

j	1	2	3	Σ j
t_j	900	1250	2110	4260 hours
r_j	1	1	1	3

By accepting a time interval for testing $t^* = 2400$ hours, according to relation (15), $\tilde{D}(\mathrm{t}) = 7020$ hours.

Observations: The testing is performed by the supplying company in the beneficiary's full or partial presence. The results of the testing are written in an official document which is given to the purchaser. The supplier is liable for different justified nonconformities. According to value $\tilde{D}(\mathrm{t})$, the beneficiary may prefer a certain supplying company.

5. CONCLUSION

The paper aimed to provide the possibility to know the behaviour under exploitation of certain pieces of equipment with modest operational parameters, but with an important role in the achievement of the basic equipment's good functionality.

The operative interventions of these components, especially when failures occur at the level of basic equipment, underline the extremely important role provided by the "starter" electro-motors.

Therefore, a thorough and unprejudiced analysis may provide a real assessment of these components' importance, in what concerns the achievement of a good availability of any transport means tributary to roads.

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