

# BUILDING SOLAR EXPOSURE SIMULATION IN THE NORTH WESTERN PART OF ROMANIA

IONESCU GH. C., BALOTA R. S., LOLEA M., BARLA E.

University of Oradea, Universităţii street No.1, Oradea,

gheionescu@gmail.com, raulbalota@gmail.com, mlolea@uoradea.ro, ebarla@uoradea.ro

**Abstract** –The simulation of the exterior meteorological conditions and its action upon the buildings, is a challenge for which the engineering software developers try to create the right software tools. COMSOL is one of the most complete and complex analysis software package available nowadays on the market. This software package is based on many modules which come to help researchers in a great variety of physics and chemistry fields. Starting with version 4.3, the module which simulates and solves heat transfer problems, has a new function: solar radiation simulation. Through this new function, architects, engineers and researchers, are now able to simulate the action of the solar radiation. In our paper, this function is used to simulate the solar radiation exposure of a one storey high building, positioned from the geographical point of view in the North Western part of Romania. The study was done for a typical summer day and it shows, in a graphical and easy to understand way, which are the most exposed areas of the building to the solar radiation. This way, we can identify the interior zones in which the problem of thermal accumulation due to solar radiation, are most likely to disturb the interior comfort parameters.

**Keywords:** radiation, emissivity, absorptivity, solar, building, North Western, Romania, COMSOL

## 1. INTRODUCTION

The new generation of software tools which use finite element methods are bringing new possibilities of physical and chemical processes simulation which happen in all our environment. Along with the increase of the availability of computing power, resulted from the alert advance of technology, these programs are now capable to calculate and simulate more accurately the complex phenomena around us.

Solar radiation is one of the phenomena which was not very easy to simulate on the computer. Researchers used complex equations, to be able to get correct results. In our study, we will use the latest version of the finite element method software package, Comsol 5.0 (demo license), to simulate solar radiation [3]. The solar radiation function was embedded starting with version 4.3 and can be found in the heat transfer module.

## 2. DEFINITIONS

**Absolute black body** (called also black body) is in physics a model for the electromagnetic energy radiant systems. The necessity of defining this concept has come up during the study of the interaction processes between the radiation and the material (emission and absorption of the radiations) [1, 2].

By definition, a black body is a body which integrally absorbs radiation without reflecting or transmitting even a fraction of the energy of the incident radiation. The black body can emit radiation. The electromagnetic spectrum of the black body emitted radiation depends only of its absolute temperature. In nature, this type of object does not exist. The definition only represents a limit situation (complete absorptivity) of the emission-absorption processes. This concept was created by the German physicist Kirchhoff in 1860. The etymology of the word suggests that a body will appear as black or absolute black because now ray of light will be reflected by its surface. By difference, a black body, which does not exist, the spectral distribution of the black body radiation, can be frequently seen in nature anytime the mater finds itself in an energetic balance with the radiation. This aspect explains why the study of the black body radiation has drawn physicist's attention. The theoretical research on the interaction of the black body with the electromagnetic radiation and the complete description of the energetic distribution in the black body radiation spectrum by Max Planck (1900), had led to the idea of quantification of the energetic exchange between radiation and materials, which contributed fundamentally to further development of quantic mechanics.[1,2,5]

**Emissivity** (emission power)  $E_M$  of a M material is the quantity of electromagnetic energy radiated with wave length between the interval  $\lambda, \lambda + \Delta \lambda$  emitted on time unit by an area element  $dA$  of the material with a  $n$  normal and a solid angle  $d\Omega$  around a direction  $n^1$  given by the angles  $(\theta, \phi)$  [1,2,5,6,7].

$$E_M(\lambda, \theta, \phi, T) = \frac{d^4 E}{dtdA \cos \theta d\Omega d\lambda}$$

It depends on the absolute temperature  $T$  of the material and can depend also on the chosen point on the body's surface. The emissivity is linked to the absorptivity (power of absorption) and reflectivity of the material through Kirchhoff's laws [1,2,5,6].

$$\frac{E_M(\lambda, \theta, \phi, T)}{A_M(\lambda, \theta, \phi, T)} = I(\lambda, T)$$

Where  $A_M$  is the absorptivity, and  $I(\lambda, T)$  is a material independent function called the black body radiation. Sometimes, the fraction below can be used to express emissivity [1,2,5,6,7]:

$$\varepsilon_M(\lambda, \theta, \phi, T) = \frac{E_M(\lambda, \theta, \phi, T)}{I(\lambda, T)}$$

This is a number between 0 and 1, equal to 1 only when  $A_M=1$  ("black body"). In the visible domain, the emissivity is also called the glow of the material. By integrating the wave lengths we obtained the integral directional emissivity and by integrating the angles we obtain the total emission power, which depends only on the temperature and the material. For a total absorbing body (absolute black body) Stefan's law is in force:

$$E_M = \sigma \cdot T^4$$

Where  $\sigma = 5,67 \cdot 10^{-12} \text{ W}/(\text{cm}^2\text{K}^4)$  ( see Wien's displacement law's) [5,6,7,8].

We can say that a material emits according to Lambert's law if  $E_M(\lambda, \theta, \phi)$  does not depend on the angles  $\theta$  and  $\phi$ . The emitted light flux under  $\theta$  angle is in this case:

$$\frac{d^4E}{dt} = E_M(dA \cdot \cos\theta)d\Omega d\lambda$$

Because the human eye assesses the glow of an object by comparing the light energy received from the apparent surface ( $dA \cos \theta$ ) of the emitter, an object which emits by Lambert's law seems to emit just as much light, independently from the watchers point of view. Particularly, a spherical object of "Lambertian" type, which emits isotropic light, seems from distance like a uniform lighted disc. The sun is a non Lambertian body because its glow is more intense in the center than on the edges.

**Reflectivity** of the surface of an M material is the fraction of the incident solar electromagnetic radiation which is reflected by the surface. For a perfect plane surface, an incident monochromatic wave is partially reflected like a mirror: the direction of the propagated wave is contained between the incidence direction plane and the normal plane on the surface. The angles of the 2 directions with the normal plane on the surface are equal. Many of the reality found surfaces are not smooth and have irregularities: an incident wave projected on these surfaces is partially absorbed and partially disseminated in all directions. To understand these surfaces, we must use a more complex definition of reflectivity. In order to do this we consider a light beam of incident rays with the opening  $d\omega$  on a generic surface element  $dA$  having the normal  $n$  starting from direction  $n_1$  given by the angles  $\theta, \Phi, n_1 n = -\cos \theta$ . The scalar multiplication between the considered direction and the normal to the surface element. The energy which falls in the time unit on the area  $dA$  is characterized by the intensity  $I(\lambda, \theta, \Phi)$  (the light beam contains wave lengths between  $\lambda$  and  $(\lambda + d\lambda)$  [1,2,5,6,7,8]:

$$d^4E = I(\lambda, \theta, \phi) dt dA \cos\theta d\omega d\lambda$$

The reflected energy by the surface element  $dA$  under a solid angle  $d\omega_r$  around the direction  $n_r$  given by the angles  $\omega_r, \Phi_r$  is:

$$d^4E_r = I_r(\lambda, \theta_r, \phi_r, \theta, \phi) dA dt \cdot \cos\theta_r d\omega_r d\lambda$$

The intensity  $I_r(\lambda, \theta_r, \phi_r, \theta, \phi)$  is proportional with the incident light flux:

$$I(\lambda, \theta_r, \phi_r, \theta, \phi) = \rho(\lambda, \theta_r, \phi_r, \theta, \phi) I(\lambda, \theta, \phi)$$

The coefficient  $\rho(\lambda, \theta_r, \phi_r, \theta, \phi)$  is called reflectivity (double directional) of the surface. It depends on the materials temperature [5,6,7,8]. The remarkable property of this phenomena is its symmetry when reported to the 2 pairs of angles:

$$\rho(\lambda, \theta_r, \phi_r, \theta, \phi) = \rho(\lambda, \theta, \phi, \theta_r, \phi_r)$$

Knowing  $\rho$  we can calculate the total reflected energy from an incident wave which comes on the direction given by the angles  $\theta, \phi$  on the element  $dA$  by integrating the over the angles  $\theta_r, \phi_r$  [5]:

$$d^4E = \left( \int \rho(\lambda, \theta, \phi, \theta_r, \phi_r) \cos \theta_r d\omega_r \right) I(\lambda, \theta, \phi) \cos \theta d\omega dt dA d\lambda$$

$$R_i = (\lambda, \theta, \phi) I(\lambda, \theta, \phi) \cos \theta d\lambda d\omega dA dt$$

This reflection coefficient  $R_i$  (i comes from incident) has the following property:

$$R_i(\lambda, \theta, \phi) + A(\lambda, \theta, \phi) = 1$$

Where  $A(\lambda, \theta, \phi)$  is the absorptivity of the surface. In the context of Kirchhoff's radiations laws  $R_i$  is called reflectivity. Alternatively we can lighten the surface from all directions and calculate the quantity of reflected energy in the direction given by the angles  $(\theta_r, \phi_r)$ . If the illumination is isotropic (this means  $I$  is independent of  $\theta, \phi$  then we define

$$R_r(\theta_r, \phi_r) = \frac{I_r(\theta_r, \phi_r)}{I}$$

(r indices comes from reflection) we verify that as a consequence of the symmetry of the function  $\rho(\lambda, \theta_r, \phi_r, \theta, \phi)$  for any pair of angles  $\theta, \phi$ :

$$R_r(\theta, \phi) = R_i(\theta, \phi)$$

Generally the formula is not valid. We can say that a surface reflects according to Lambert's law if the function  $\rho(\lambda, \theta_r, \phi_r, \theta, \phi)$  does not depend at all on the variable set,  $\theta_r, \phi_r, \theta, \phi$ . A plane object, which reflects according to Lambert's law, illuminated under a fixed exterior angle, seems to glow at the same intensity,

independent from the point of view. A uniformly illuminated sphere which reflects according to Lambert's law, must have a luminosity which tends gradually (like  $\cos \theta$ ) to 0, when the incident light ray becomes tangent to it ( $\theta \rightarrow \frac{\pi}{2}$ ). Taking the moon for example, the passing between light and obscurity is done suddenly. This shows that this object is not a "lambertian" object. By using the symmetry of the function  $\rho(\lambda, \theta_r, \phi_r, \theta, \phi)$  we conclude that, at a normal incidence, the reflectivity at big  $\theta_r$  angles has also big values.

If the surface is smooth, its reflectivity is described by on only function (for un-polarized light)  $R(\theta)$ . This can be calculated with the help of Maxwell's equations, knowing the refraction indices and the absorption coefficients of the both media which a separated by the surface (Freshnell's Formulas).

**Absorptivity** (Absorption power)  $A_M$  for the surface of a M material is the fraction of the electromagnetic energy with wavelengths in an interval  $\lambda, \lambda + \Delta \lambda$ , incident in the time unit over a dA surface element with the "n" normal under a solid angle  $d\Omega$  around the n1 direction given by the angles  $\theta, \phi$   $n_1 \cos \theta$ . The scalar multiplication of the normal of the surface and the considered direction, which is absorbed by the surface [5,6,7].

$$d^4 E_A = A_M(\lambda, \theta, \phi, T) d^4 E_I$$

$$\equiv A_M(\lambda, \theta, \phi, T) I(\lambda, \theta, \phi) \cos \theta dA dt d\Omega d\lambda$$

Where:

$d^4 E_A$  Absorbed energy

$d^4 E_I$  Incident energy

$I(\lambda, \theta, \phi)$  Intensity of the incident radiation

The absorptivity depends on the T temperature of the M material and may differ between the points of the surface. Absorptivity can be represented as a real number contained between 0 and 1. [1,2,5,7] An imaginary body whose surface has the absorptivity equal to 1 for al the incident angles and for all the wavelengths is called a black body. The number 1-A is called reflectivity (reported to a given incident direction):

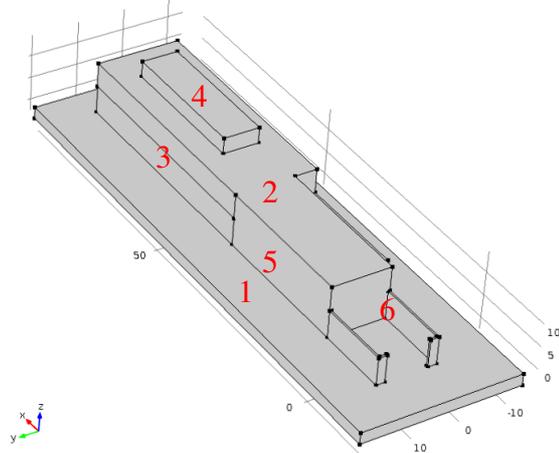
$$R_i(\lambda, \theta, \phi, T) + A_M(\lambda, \theta, \phi) = 1$$

Absorptivity is connected to emissivity through Kirchoff's laws. Absorptivity is a property of the surface and must not be confused with the absorption coefficient (attenuation) of the radiation during its propagation process through media. To show a possible confusion, we will present the following example: the surface of a metallic object has high reflectivity and of course low absorptivity, although the attenuation (absorption) of an electromagnetic wave inside the material is high, by giving away energy to free electrons.

### 3. GEOMETRY OF THE MODEL

The 5<sup>th</sup> versions of the well known finite element method software package, COMSOL brings new enhancements which increase the interoperability of the program with known (Inventor, Solidworks, Autocad,

CATIA etc.) modeling software tools on the market. For the study presented in our paper, the model is 3D, and it was designed using Autocad demo version. The geometric model is simplistic, in order to avoid the hardware demanded to calculate the solution. Our building is represented as a compact box with several surfaces (see Fig. 1)



**Fig. 1 Tridimensional view of the geometric model**

The studied building has 2 levels, a ground flow and one storey. Its destination is as an office building, but the ground floor hosts on the southern façade a workshop which has an extended height. Almost 70% of the building's area is represented by offices. It should be mentioned that for this study, the neighboring buildings were not taken into consideration. The neighboring building can increase the quantity of radiation sent to the studied building, through reflection. The compact plan shape of the building is polygonal.

### 4. MATERIALS AND THEIR PHYSICAL PROPERTIES

The studied building's envelope has a significant glazed area which summed up reaches a lateral area of 862, 6 m<sup>2</sup>. This area consists of curtain walls (figured with 3 in Fig. 1) with aluminum structure, and big glazing panels having the following dimensions h=4,45m and b=1,45m. Another portion of the facades is covered with 5cm thick ruukki sandwich panels with polyurethane foam core. This area is of 506, 8 m<sup>2</sup> – signed with no.5 in Fig. 1

An important element of the buildings envelope is the flat roof (indicated with no.2), because, as we will see further, this zone is the most exposed to solar radiation. The terrace roof is composed of multiple layers, and for this particular building the top layer is composed of 2 black colored bitumen waterproof insulation layer. For this reason this area absorbs an important heat quantity and transmits it toward the interior layers, bringing its contribution to the amount of gained heat from exterior sources.

On the roof, the HVAC generation unit can be seen (no.4 in Fig. 1). This installation is mounted inside in a enclosure of sandwich panel with polyurethane core between galvanized metallic sheets. A buffer zone (noted

with no.6) for industrial auto access to the embedded workshop is position on the southern facade of the building. Its height extends to the first floor. The surrounding walls are made of concrete blocks. [6,7,8]

Around the building is a concrete platform.

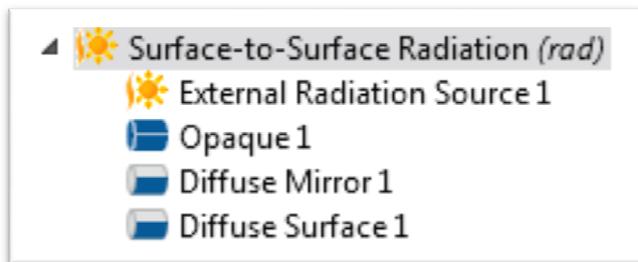
**Tab.1 Material characteristics**

ID	Material	Emissivity
1	Concrete ground floor	0,94
2	Bitument waterproof insulation	0,86
3	Glass curtain walls	0,94
4	Polyurethane sandwich panel with metallic sheets	0,28
5	Polyurethane sandwich panel with metallic sheets	0,28

**5. INPUT DATA**

Comsol has multiple functions integrated within the heat transfer module. For our model we chose Heat Transfer with Surface to Surface Radiations in

Fig. 2. [3,4]



**Fig. 2 Interface input data selection**

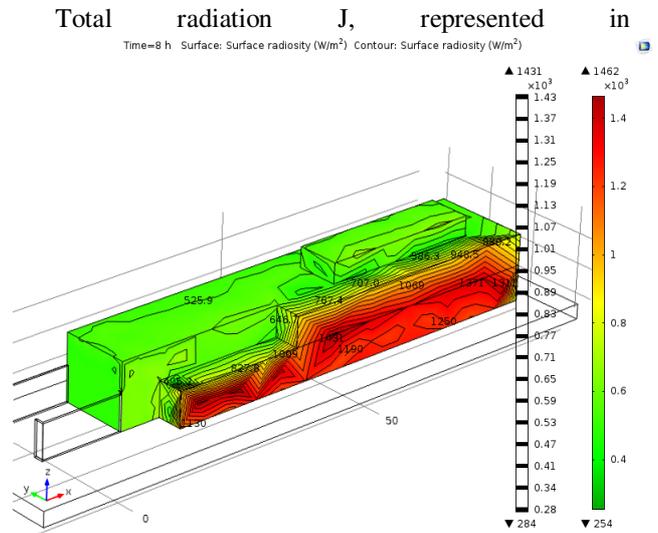
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**Fig. 3** the function control interface for the solar radiation is presented. First we need to introduce the source of the radiation, which in our case is the solar radiation. Then the position of the sun must be established. This operation is done by mentioning the latitude and longitude number of the studied location. For the North Western part of Romania, our coordinates are 47° Latitude and 23° Longitude. In order to accurately calculate the sun's trajectory for the studied interval, the time zone for the studied location must be entered. In the date fields, we must input the date for which is performed the simulation. For our study we used a generic summer day 01 July 2012. In order to establish the exact position of the sun, also the time of the day must be inserted in the correct fields. Our study is time dependent, which is why we inserted the "t" variable in the "Second" field. The "t" variable is the time counter which makes our radiation source (the sun) move. Comsol increases the value of this variable by seconds.

The value of the solar radiation was considered 1000 W/m<sup>2</sup>.

**6. RESULTS AND DISCUSSIONS**

**6.1. Received radiation**



**Fig. 9** is the sum of the reflected radiation and the emitted radiation.

In

**Fig. 5 and**

**Fig. 6** are graphically represented through specific colored surfaces and isolines, the values of the received solar radiation on all the external surfaces of the building. These values are calculated for 08.00 AM and for 18.00.

### External Radiation Source

Label:

Equation

Show equation assuming:

$$G_{ext,Bi} = F_{ext,Bi} \cdot (i_s) \cdot q_{0,s} \cdot FEP_{Bi}(T_{sun})$$

$\vec{x} \equiv North, \vec{y} \equiv West, \vec{z} \equiv Zenith$

External Radiation Source

Source position:

Location defined by:

Location

Latitude (+ to	Longitude (+ to	Time zone (+UT
47	23	2

Date

Day	Month	Year
01	06	2012

Local time

Hour	Minute	Second
0	0	t

Solar irradiance:

$I_s$    $W/m^2$

Fig. 3 Solar radiation function control panel

In order to simulate the evolution of the exterior temperature, we used a first grade function, which is represented in the graph seen in Fig. 4.

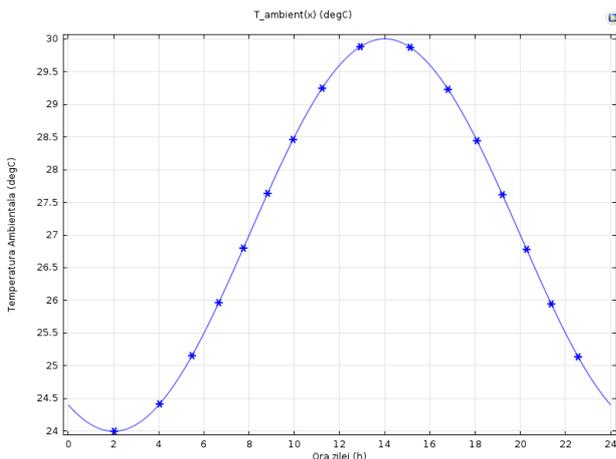


Fig. 4 Exterior temperature values for the generic summer day considered in our study

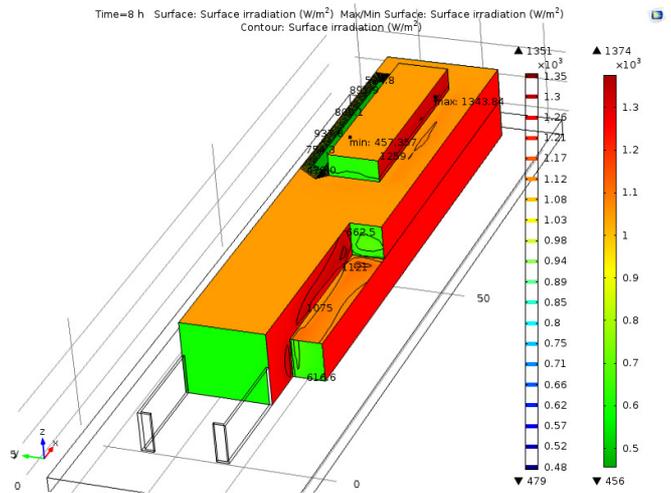


Fig. 5 Total radiation received by the building's envelope at 08.00 AM.

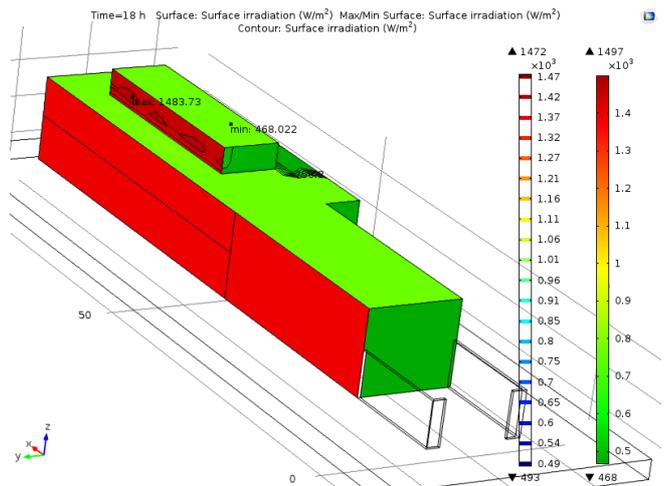
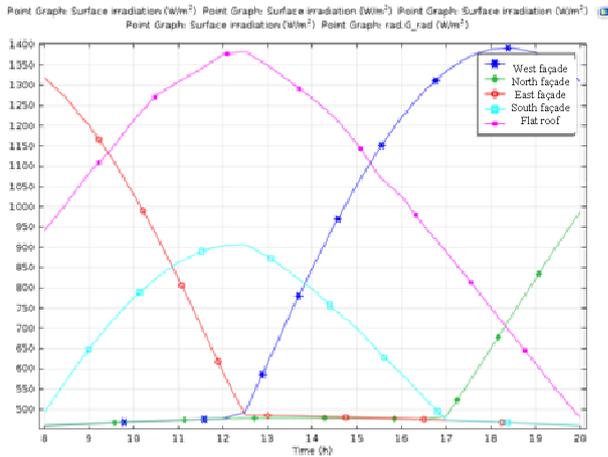


Fig. 6 Total radiation received by the building's envelope at 18.00



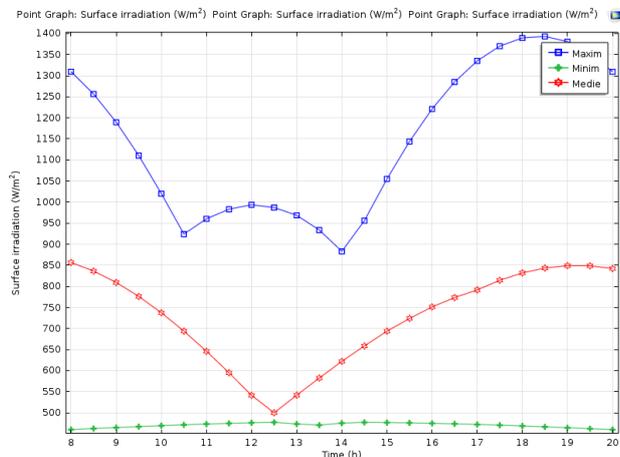
**Fig. 7 Daily evolution of the solar radiation received on the 4 facades of the building and the flat roof.**

We can observe in **Fig. 7** the evolution of the values for the received solar radiation received on the 4 facades of the building and the terrace roof along the studied day. As expected, the most important quantity of solar radiation was received on the terrace roof. The amount of radiation is around 1000 W/m<sup>2</sup> at 08.00 AM, then reaching the maximum value of 1350 W/m<sup>2</sup> at 12.00 AM. The minimum value is reached at the end of the day around at 20.00 hours and has the value of 500W/m<sup>2</sup>.

When studying the facades, we observe the fact that the north façade has a reduced solar exposure, when comparing it to the remaining facades. This façade receives the maximum solar radiation (approx. 1000 W/m<sup>2</sup>), only by the end of the day between 17.00 and 20.00.

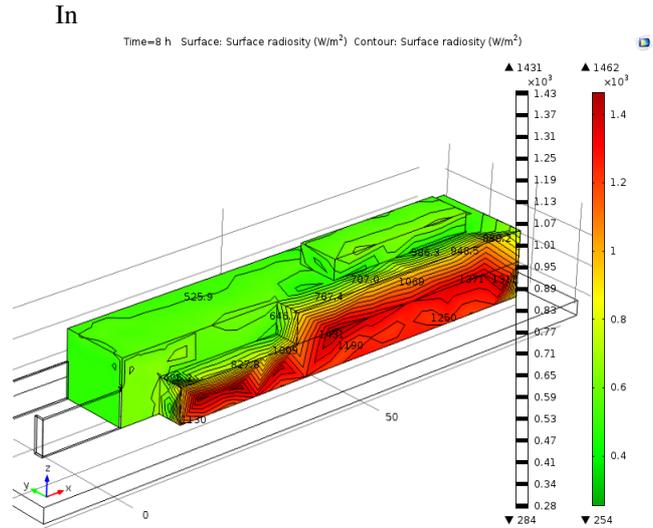
The eastern façade is mostly exposed in the morning, getting the maximum value of (1275 W/m<sup>2</sup>) at 08.00 AM. The southern façade is not affected by solar radiation due to the shape of the building. This façade is shaded by the exterior walls present in the workshop access area.

The western façade has the most significant solar exposure, during the specified summer day. Starting with 12.00 and until 18.00, this façade receives a maximum of 1400 W/m<sup>2</sup>. After 18.00 the value of the solar radiation begins to drop (see **Fig. 8**).



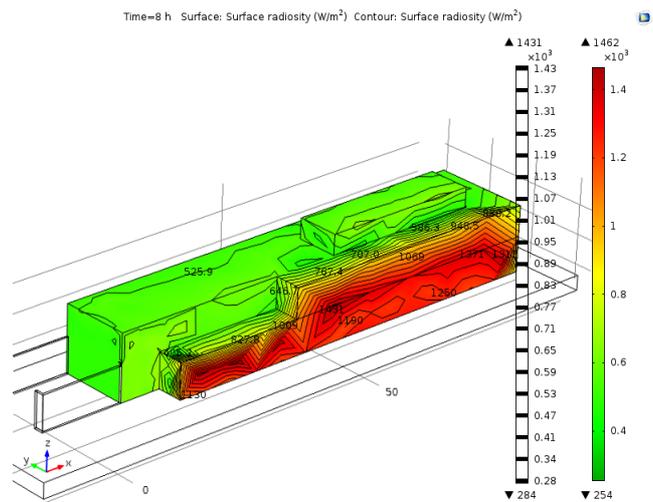
**Fig. 8 Minimum, medium and maximum Solar radiation on the glazed facades daily evolution**

**6.2. Emitted radiation**

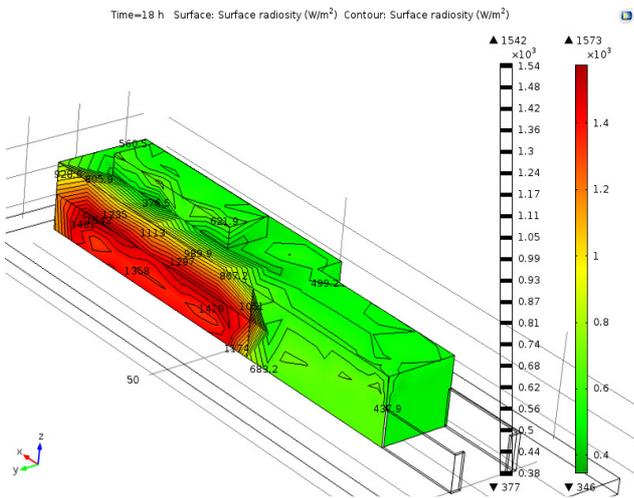


**Fig. 9** and **Fig. 10** you can find a graphical representation of the values of emitted solar radiation by the building's envelope elements. We can see that the biggest amount of radiation is emitted by the curtain walls covered area. The value 1462 W/m<sup>2</sup> of the emitted radiation can be found on the eastern façade at 08.00 AM. At 18.00 the value is 1573 W/m<sup>2</sup>. In **Fig. 11** we can observe the maximum, medium, minimum emitted solar radiation by the curtain walls for the studied day, for the curtain walls covered area of the building's envelope. In the mentioned graph, we can see that the maximum emitted solar radiation is higher in the afternoon. Having an area of 862, 6 m<sup>2</sup> of curtain walls we can calculate the total amount of emitted solar radiation by the glazed façade.  $A_{cw}=862, 6 m^2$ .

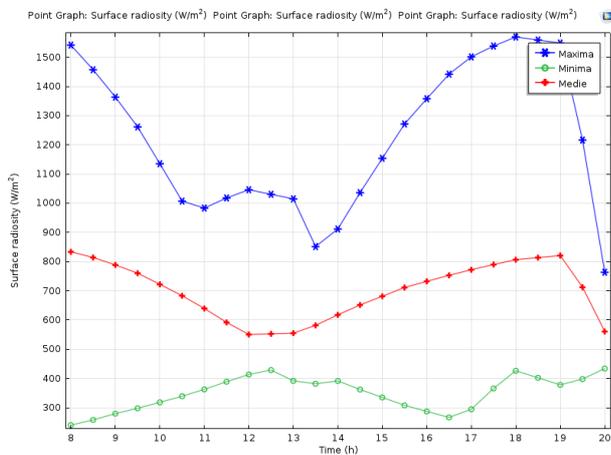
$$E_{r_{cw}} = A_{cw} \cdot 1573 \frac{W}{m^2} = 1356869.8 W = 1356.9 kW$$



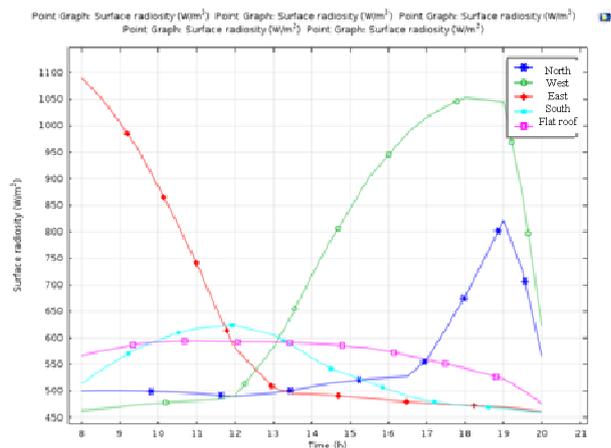
**Fig. 9 Total emitted radiation on the exterior surfaces of the building's envelope at 08:00**



**Fig. 10 Total emitted radiation on the exterior surfaces of the building's envelope at 16:00**



**Fig. 11 Maximum, average, minimum emitted solar radiation by the curtain walls for the studied day**



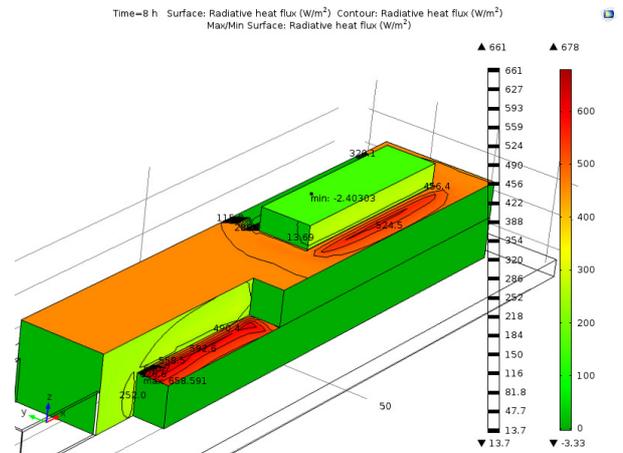
**Fig. 12 Daily evolution of emitted solar radiation for the 4 facades and the terrace roof.**

### 6.3. Transmitted heat flux to the inner layers of the building's envelope

The transmitted heat flux is calculated by making the difference between the received radiation and the emitted

radiation on the external surfaces of the envelope. In **Fig. 13** and

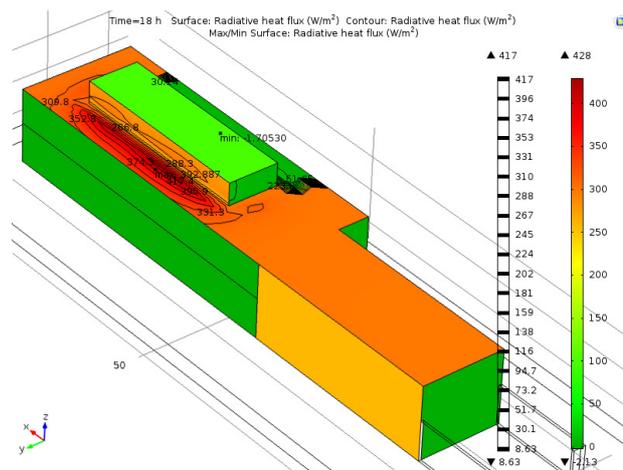
**Fig. 14**, we can see that naturally the glazed exterior surfaces of the building are reflecting a big amount of the received solar radiation.



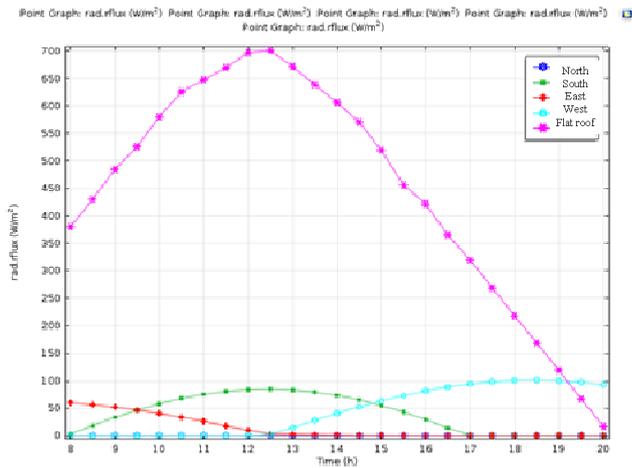
**Fig. 13 Transmitted heat flux to inner layers of the building envelope at 08.00**

The surfaces of the sandwich panels or the flat roof are collecting a part of the solar radiations into a heat flux which is then transmitted to inner layers of the envelope's element.

In **Fig. 15** we understand that the biggest heat flux to inner layers is transmitted through the roof of the building. This happens because of the material used to insulate against the water infiltration the flat roof. The waterproof insulation consist of bitumen, a material which is black and which gains heat from the solar radiation. Especially during summer, this material stores an important amount of heat. According to our simulation, in the terrace roof area a quantity of  $700\text{W/m}^2$  is transmitted to the inner layers of the roof between 12.00 and 13.00. By using this information, we will be able to calculate the necessary type and thickness for thermo insulating layers.



**Fig. 14 Transmitted heat flux to inner layers of the building envelope at 18.00**



**Fig. 15** Daily evolution of the heat flux transmitted to inner layer on the 4 facades and the terrace roof

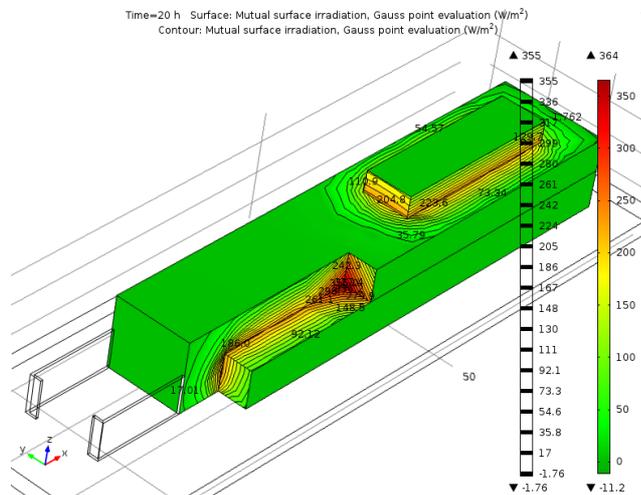
**6.4. Mutual surface irradiation**

$G_m$  is the mutual irradiation (SI unit:  $W/m^2$ ) arriving from other surfaces in the modeled geometry,  $G_m$  is determined from the geometry and the local temperatures of the surrounding boundaries. By using the information calculated with this simulation we can better distribute the type of materials on the building's envelope. In

**Fig. 16** it can be observed that the most mutual irradiated areas are at the corners of the envelope. The conclusion is that, from this point of view, the shape of the building should have as few interior corners as possible.

**CONCLUSIONS**

The exact analysis of the solar radiation quantity which is received and then some of it retained while the other part is emitted by the elements of the building's envelope, gives architects and engineers the complete view over the opportunities of optimization from the design phase. Through this kind of analysis we are able to study the influence of the solar radiation on our building in different scenarios.



**Fig. 16** Mutual irradiation at 20.00

Beside this, we can simulate and evaluate the amount of solar energy we can collect by using solar photovoltaic panels. This analysis will help us position correctly the building on the field, taking into consideration the most favorable action of the sun. In this study also evaluated the amount of radiation which this building can reflect to neighbouring buildings.

For our study we used an already build construction. For greenfield project, this kind of analysis is mandatory.

In our case, we conclude that the glazed area, which occupies an important percentage of the building's envelope, is highly exposed to solar radiation and needs to be optimized from this point of view. The northern facade is not exposed, and represents a weak point of the building from the thermal point of view. Taking in consideration this simulation's resulted graphs we can propose methods for the enhancement of:

- the materials used to create the building's envelope,
- the shape of the building
- natural ventilation system
- solar energy collection system

Taking into account that Romania has a medium of 210 sunshine days per year which means 57%, this kind of simulations and studies need to be done especially on curtain wall closed buildings. In our case we will simulate enhancements in the shape of the building, in order to improve the natural ventilation. The natural ventilation system could consist of small closable holes left in the curtain walls, in the right places which should allow natural cooler air to enter the interior. This study can help us establish the schedule in which the ventilation holes should remain open or closed. After this analysis we conclude that in the morning cool air could enter through the western facade, which is not so exposed to solar radiation and then, in the afternoon the open holes should be on the eastern facade. This study can be completed by adding a fluid flow study.

Comsol 5.0 software presents a new generation of software tools, which can ease up the research work by simulating fast multiple physics scenarios. Using those tools together, we can get precious information about the direction to search the solutions to the technical challenges we encounter. In future papers we will study several versions of placement of a building in a surrounding environment while taking into account the effect of the neighbouring buildings upon each other.

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