Abstract The specific problems of the urban public transport system in the literature of the field are widely reflected; the main targets are to increase the efficiency, the quality of service and to reduce their impact on the environment. The electric driven transport is an alternative that may operate according to these targets. This article inscribes in the actual concerns dedicated to optimize the tram parks with which the urban public transport companies (UPTC) operate, using an optimizing criterion, which leads to an efficient and quality transport service. In this article, after evoking the importance of this concern, the mathematical model of optimization is elaborated in two variants (general and simplified). The proposed analysing model is for an applied UPTC that operates a mixed system (trams + buses), in real level of reliability conditions of trams.

Key words: urban public transport, tram park, optimization, availability, reliability.

1. INTRODUCTION

Sustainable development is one of the dominant topics of globalisation. Transport and energy are two key challenges in the sustainable development strategy of the European Union (EU) [1]. It is known, that at the EU level, the power consumption in systems of transport accounts for about 30% of all consumption, greenhouse gas emissions caused by the means of transport being as in [2]. Therefore, it is clear that environmental concerns with regards to reduction of energy consumption in the transport systems are very important and included in the main concerns to increase the social, economic and environmental performance of these systems [3, 4].

From perspective of sustainable development, it is essential for the local authorities to take care that:
- Development takes priority over urban public transport (UPT), which will reduce auto traffic;
- Development of electrified UPT, transport system that is much cleaner, is relatively quiet and enhanced safety in circulation.

The specific problems of the urban public transport systems are reflected in the literature of the field. A significant part of this work deals with the performance of UPT systems, performance quantified by: efficiency, quality of service, and impact on the environment. In [4, 5] the factors identified are those influencing the application of UPT, with a particular focus, on the quality service, and on [6] the methodology applied in the preparation of studies of quality of the UPT system proposed and exemplified. Detailed methodology for the assessment of quality of the transport service is made in [7, 8], using decisive impact factors such as: availability, comfort, and convenience. The energy efficiency of UPT with electric traction is presented in [9] and in [10], is presented a solution of energy efficiency and of electrically driven trams comfort increasing (EDT). It aims at determining energy efficiency for locomotives and its impact on the optimization of the running lines parameters, the modernisation of the stations and optimal management of traffic.

The present article evokes the above concerns about modelling and performance evaluation of the public transport systems, but it is distinguished by the fact that it is dedicated to optimum dimensioning of the tram park within the urban public transport company (UPTC) with electrically driven trams EDT. The operational aspect of EDT can be regarded and treated as a result of the interconnection of three subsystems [11, 12].

The operational studies reflect the fact, that the availability of EDT has values that may decrease up to 50% of the nominal value, depending on many factors [4]. Under these circumstances, to ensure the public service, UPTC treats and structures SSEDT as a system of “k-out-of-n” [13,14] type, where \( n \) is the total number of EDT and \( k \) is the number of EDT in use, necessary to cover the engaged services by the company. Starting from this reality and treatment mode, this article tries to establish the optimal value of all EDT \( n_{opt} \) numbers, depending on the load chart \( k = f(t) \) and availability level of EDT within SSEDT.

In this article, the author’s previous experience is harnessed regarding the optimization of system’s with “k-out-of-n” type, the previous applications [15, 16] being dedicated to optimum management of maintenance and to maximization of availabilities of such structures used within power systems.

2. THE MATHEMATICAL MODEL

Generally, the EDT park within an UPTC may be considered as an EDT of \( m \) type, with different technical characteristics and availabilities.

It can be denoted:
\( n_i \) – total number of EDT of \( i \) type
\( k_i \) – necessary number of \( i \) type EDT, to ensure the assumed service by the company.

Obviously:

\[
 n = \sum_{i=1}^{m} n_i ; \quad k = \sum_{i=1}^{m} k_i \quad (1)
\]

The problem that must be resolved is to determine the optimal value of the vector:

\[
 n_{\text{opt}} = \{ n_{1\text{opt}}, n_{2\text{opt}}, \ldots, n_{m\text{opt}} \} \quad (2)
\]

when the load chart of UPTS is variable in steps \([k\) being variable with the time \((t)\)] and the technical characteristics of EDT are different, depending on their type. To resolve this problem, from the point of view of the technical characteristics we are interested in EDT transport capacity and its time availability.

Such a problem is that of dynamic programming, applied to optimizing the reserve of the system, a problem which general solution is well known [17]. As compared to the general cases, for which the criteria of “maximizing of reliability” or “minimizing the costs” is applied, without reducing the level of reliability under a minimal value, for the analysed case the optimum value of a vector \( n_{\text{opt}} \) will be obtained by using the criterion:

\[
 A_i (n,k,A) = \sum_{j=1}^{N} f_j (n_1,k_1,A_1; n_2,k_2,A_2; \ldots; n_m,k_m,A_m) \geq A_{\text{min}}
\]

with the following restrictions:

\[
 0 \leq k_i \leq n_i \leq n_{i\text{max}} \quad i = 1,2,\ldots, m \quad (3)
\]

where:

\( A_i, A_j \) – the availability of EDT of \( i \) type \((A_i)\) and availability of SSEDT \((A_j)\);
\( A_{\text{min}} \) – a minimal imposed value for the analysed SSEDT availability;
\( n_{\text{max}} \) – maximum value for the variable \( n_i \);
\( N \) – the number of terms of calculation expression for the system availability.

By setting the value of \( A_{\text{min}} \) the fulfillment of at least two criteria from the indicators set by European Standards [3] is ensured respectively, the availability and access to service, as by identifying of \( n \) vector the minimal costs is obtained for the imposed level availability of UPT service.

The availability of which EDT type is calculated depending on the reliability and maintainability [13, 14].

\[
 A_i(t,t_M) = R_i(t) + F_i(t)\cdot M_i(t_M) \quad i = 1,2,\ldots, m 
\]

where:

\( (t, t_M) \) – random variable “mean time between failures” \((t)\) and “time for maintenance” \((t_M)\);
\( (R_i,F_i,M_i) \) – reliability \((R_i)\), non-reliability \((F_i)\) and maintainability \((M_i)\) of \( i \) type EDT

The precision is made, that values of \( R_i, F_i, M_i, A_i \) are of probabilistic type and they are determined by statistical data processing regarding their behaviour in exploitation of EDT [12, 13]. The availability of SSEDT \((A_j)\) is determined by the sum of significant terms (for the successful states) from Newton’s generalized binomial theorem [18], where the number of operating EDT is at least equal to \( k \).

Therefore:

\[
 A_j = \sum_{j \in \{S\}} \prod_{i=1}^{m} [(A_i + U_j)]
\]

where:

\( U_j \) - unavailability of EDT of \( i \) type \((U_i = 1 - A_i)\);
\{\( S \)\} – the set of SSEDT successful states is defined as the state of the available number of operating EDT, at least equal to regular, to satisfy the needs of passengers.

The problem will be resolved within this frame, allowing the following simplifying hypothesis:

H1: The analyzed \( m \) types of EDT of the park structure, are equivalent regarding their transport capacity (TC), which means, that either equal values for TC, or the supplementary values of TC for some EDT types does not influence the availability of the traffic.

H2: The values of the \((A_i, R_i, M_i)\) indicators are those of steady state, resulted from the expression of the distribution functions, which reflect with high fidelity the empirical distribution of statistical data behaviour in exploitation of EDT of \( i \) type.

H3: The algorithm is applied for the maximum value of \( k \) variable, justified by the fact that the sizing of the EDT park is a long term action (years), and the load diagram \( k = f(t) \) is available for short term (day, week). With \( k_{\text{max}} \) is denoted the maximum value of variable \( k \) if the peak load is that in figure 2, 3 (example, for the presented study case).

Remark: for a new EUTS the values of \((A_{\mu}, R_{\mu}, M_{\mu})\) indicators are obtaining from the EDT supplier.

In these conditions, the optimization algorithm is applied in the following steps:

S1: Based on the expressions of \((R_{\mu}, M_{\mu})\) indicators, for allowable values of \((t, t_M)\) variables are calculating the values of the availabilities \((A_i), i = \bar{1}, m\).

S2: The obtained values of \( m \) at \( S_1 \), are ordered in a variation string \((S_A)\) and the correspondence with \((C_A)\) is evidenced:

\[
 S_A : \quad A_1 \geq A_2 \geq \ldots \ldots \geq A_i \geq \ldots \ldots \geq A_m 
\]

\[
 (C_A) : \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow 
\]

\( n_1, n_2, \ldots, n_i, \ldots, n_m \)

\( S_3 \): The number of EDT is summed, that is necessary to ensure the assumed service of UPTC at the maximum level of \( k_{\text{max}} \) going through \( S_3 \) in decreasing sense of \( A_i \) values;

Supposing that is obtained:

\[
 k_{\text{max}} = \sum_{i=1}^{\bar{m}} n_i + k_e 
\]

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Therefore, to cover the peak load, \( k_{\text{max}} \), there are used the first \("(\varepsilon - I)"\) types of EDT and a number of \( k_{\varepsilon} \) EDT of \( \varepsilon \) type.

\( S_C \): For the whole EDT is calculating \( A_\varepsilon \) and it is compared with \( A_{\text{min}} \), so:

- a) If \( A_\varepsilon \geq A_{\text{min}} \), the algorithm stops and we have:

\[
A_{\text{opt}} = \{n_1, n_2, \ldots, k_\varepsilon, 0, \ldots, 0\} \tag{9}
\]

- b) If \( A_\varepsilon \leq A_{\text{min}} \), the number of EDT of \( \varepsilon \) type is incremented with 1;

\( S_S \): the restrictions (4) and condition (3) is verified, so:

- a) If any restriction is exceeded, then go to \( S_6 \);
- b) If no restriction is exceeded, then go to \( S_5 \);
- c) If condition (3) is satisfied, then go to \( S_8 \).

\( S_S \): The vehicle (EDT) introduced to \( S_{\text{res}} \), is eliminated, if all these types of exemplars are exhausted;

\( S_S \): The number of EDT in reserve is incremented with \( I \), going through successively the variation string from \( \varepsilon \) type to \( m \) type;

\( S_S \): The value of \( A_\varepsilon \geq A_{\text{min}} \) is calculated and the optimum allocation is stabilized, namely \( n_{\text{opt}} \).

A simplified case is when EDTs within UPTC, are only of one type, or, they work with one equivalent type with \( A \) availability. In this case, the availability of the system in relation with the maximum load \( k_{\text{max}} \) and the total number of EDT \( n \) is with the following relation calculated [18]:

\[
A_\varepsilon = \sum_{j=k_{\text{max}}}^{n} C_n^j A_j^\varepsilon (1-A_j)^{n-j} \tag{10}
\]

where \( C_n^j \) is the combination of \( j \) units from the total of \( n \) units.

In this case, the applying of the optimization algorithm is more simplified, because is no more necessary to follow all the above steps, only the followings:

\( S^2_5 \): It is calculated \( A_\varepsilon \) with relation (10), for the \( n \) existent EDT, or planned for the UPTC endowment;

\( S^2_5 \): It is compared with the obtained value of \( A_\varepsilon \) obtained with the planned limit \( A_{\text{min}} \), so:

- a) If \( A_\varepsilon = A_{\text{min}} \), then \( n = n_{\text{opt}} \);
- b) If \( A_\varepsilon > A_{\text{min}} \), it will be made a successively decrementing with \( I \). The decrementing is used starting with the \( n \) value.
- c) If \( A_\varepsilon < A_{\text{min}} \), it will be made a successively increamenting with \( I \) of the reserve EDT, until \( A_{\text{min}} \) will be attained, so stabilizing \( n_{\text{opt}} \).

Another simplification would be, when it works with the indicator of reliability function of EDT \( (R) \) and not with availability \( (A) \). This simplification is justified only when maintainability \( (M) \) is not expressed. In this case, the results are more optimistic, in the sense that for the same SSED T for variable \( n_{\text{opt}} \) lower values are obtained, as in the case of the operation with indicator \( A \). For equivalence it is necessary, as it operates with indicator \( R \), for obtained values for \( n_{\text{opt}} \) indicator, the number of EDT in maintenance must be summed, according to existent statistics by UPTC.

Besides the evidence of the principal values of \( n_{\text{opt}} \), within a concrete analyze it is useful to graphically evidence:

\[
A_\varepsilon = f(t), R_\varepsilon = f(t), n_{\text{opt}} = f(t_{\text{in}}), n_{\text{opt}} = f(k), n_{\text{opt}} = f(t)
\]

Examples regarding the respective graphs will be presented in the following part of the paper.

3. CASE STUDY

The algorithm was applied to “Oradea Transport Local” (OTL) company, which manages EDT in Oradea municipality – Romania. The main characteristics of EUTS within OTL are:

- Double rail length divided into five zones: 39.86 km.
- The total EDT: 73 of which, for maximum carrying capacity 44 pieces are required.
- Types of EDT used:
  - Produced by TATRA (Czech Republic), with DC motors without energy recovery, type KT4D (20 pieces) and T4D + B4D (43 pieces)
  - Produced by SIEMENS (Austria) ULF (Ultra Low Floor): 10 pieces.
- Average number of daily served passengers: 22,761.
- Average travelled distance 2,405,000 km a year;
- Average cost of operation: 4,956 thousand Euro.

The weekly ranked load characteristic of EDT park is given in figure 1, and for a peak day (Wednesday) in figure 2.
The maintenance strategy applied within OTL is a mixed one (preventive and corrective). By sizing the EDT park of OTL, we must take into account the fact that the three EDT in PM are considered unavailable, as the function of the maintainability that enters the expression of the availability refers only to actions of corrective maintenance.

Table 1. Expressions of distribution function for random variables $t, \ t_{CM}$

<table>
<thead>
<tr>
<th>Type</th>
<th>Expression of reliability function $(R)$</th>
<th>Expression of corrective maintenance function $(M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ULF(1)</td>
<td>$R_1(t) = e^{-t/298}$</td>
<td>$M_{1{t_{CM}}}(t) = 1 - e^{-t/9.95}$</td>
</tr>
<tr>
<td>CT4D(2)</td>
<td>$R_2(t) = e^{-t/92.6}$</td>
<td>$M_{2{t_{CM}}}(t) = 1 - e^{-t/92}$</td>
</tr>
<tr>
<td>T4D B4D(3)</td>
<td>$R_3(t) = e^{-t/81.4}$</td>
<td>$M_{3{t_{CM}}}(t) = 1 - e^{-t/14.5}$</td>
</tr>
</tbody>
</table>

By processing the obtained statistical data from behaviour of EDT in exploitation of OTL, for period 2006 – 2011, we also obtained the functions of distribution $(t, \ t_{CM})$ that shape with the least error the empirical distribution of the two random variables \[17,18\]. These expressions are in Table 1 given. Therefore, for the three types of EDT the concrete expression of availabilities are obtained basing on the general expressions (5).

In the first phase, the optimization algorithm with $A_{\text{min}} = 0.99$, in the simplified variant was applied, if working with an equivalent EDT with indicator values $(R, A)$ determined by the relations:

$$R_i(t) = n_1 R_1(t) + n_2 R_2(t) + n_3 R_3(t)$$
$$A_i(t, t_{CM}) = n_1 A_{1\{t_{CM}\}} + n_2 A_{2\{t_{CM}\}} + n_3 A_{3\{t_{CM}\}}$$

The obtained results are presented by diagrams 3 ÷ 8.
4. DISCUSSION

From the expression of “reliability function” indicator (table 1), it results that trams of KT4D and T4D type are in the third period of utilization – stage of accelerated wearing, the intensity of faults exponentially increases with time \( \beta_2 = 1.135 \) – for KT4D and \( \beta_2 = 1.05 \) – for T4D. The tram of ULF is in the first utilization period, at the end of the running interval, the fault intensity is decreasing, being equivalent with the increase of reliability \( \beta_1 = 0.97 \).

From the shapes of figures 3 to 7, we mainly find that:

- The reliability of the analyzed park (SSEDT-OTL) is maintained, practically, at an ideal value \( (R = 1) \), for operating time \( t \) without maintenance for 35 hours \( (t \leq 35 \text{ hours}) \). Beyond this value of operating time, works of preventive maintenance must be applied in the tram park, because its reliability decreases rapidly (figure 3). By a successive preventive maintenance for EDT within SSEDT-OTL, its availability may be maintained at a pre-established value, \( A_i \geq 0.99 \);
- As it is expected, the reliability of the tram park will increase if the number of EDT in reserve increases (figure 4) and decreases with increasing of the using period without maintenance;
- The availability of the analyzed tram park (SSEDT-OTL) increases with the profoundness increasing of the corrective maintenance work, which implies an increase in the duration of this work (figure 5). Reasonable values for availability of such kind of systems (\( A_i \geq 0.99 \)), are obtained for \( t_{CM} \geq 14 \text{ hours} \) and \( t \leq 300 \text{ hours} \). For quick corrective maintenance, superficially \( (t_{CM} < 14 \text{ hours}) \) there is a drastically reduction of the availability of tram park at reasonable continuous operation duration between two maintenance works \( (t \geq 100 \text{ hours}) \).

Applying the model of proposed optimization within OTL, for equivalent tram, the followings can be found (figures 6, 7):

- The optimum number of trams \( n_{opt} \) strongly depends on the duration of the corrective maintenance work \( (t_{CM}) \), \( n_{opt} \) increasing with the increase of \( t_{CM} \) value;
- The reliability of the trams, respectively, the time between two failures \( t \), has an important impact on \( n_{opt} \) variable value;
- The influence of the two random variable \( (t, t_{CM}) \) on \( n_{opt} \) value blurs over certain values, respectively over \( t = 800 \text{ hours} \) and \( t_{CM} = 20 \text{ hours} \);
- The maximum loading level \( (k_{max} = 44) \), at an equivalent reliability and maintainability, is translated by \( t = 100 \text{ hours} \); \( t_{CM} = 10 \text{ hours} \) and three trams (being in preventive maintenance) is the number of actual trams \( n_{opt} = 76 > 73 \) obtained.

5. CONCLUSION

Public urban transport is an activity with great responsibility in towns and cities and in the same time, a concern with important actuality. Special companies make urban transport systems as adequate operation, which aims in equal measure efficient and quality services realizing. Urban public transport with electric driven trams is a variant imposed through the prism of advantages under the aspect of environment impact, passenger comfort and possibilities to harness renewable energy sources.

Tram park optimization with which the urban public transport companies operate is a way to increase energy and economic efficiency of such transport system, while insuring the necessary level of availability and quality of urban public transport service. To stabilize the optimum number of trams \( n_{opt} \), in an urban public transport company, the ensemble (park) of trams is treated as a system of “k-out-of-n” type, aiming to stabilize the optimum value of the variable \( n (n_{opt}) \) depending on the load chart \( k = f(t) \) and on level of the tram operational availability. The applied criterion in this paper is to size the optimum tram park, to “ensure the minimum level imposed to the system’s availability” taking into account the real level of the tram’s reliability and maintainability.

OTL Company, for which this study is realized, is a medium company and manages a mixed urban public transport system with three types of trams and buses. The optimizing model was applied for the tram park within OTL Company, working with equivalent tram. It appears that the values of variable “optimum number of trams” within the company \( n_{opt} \) are strongly depending on random variable values “time between failures” and “corrective maintenance time”.

Actual reliability level of the three tram types, implies a differentiated number of reserve to satisfy the fixed optimizing criterion \( A_{min} = 0.99 \), which is by following \( n_{opt} \) variable values translated for \( k_{max} = 44 \), with hypothesis assuming that there are sufficient trams of each type to cover the necessities.

<table>
<thead>
<tr>
<th>Tram type (EDT)</th>
<th>ULF (1)</th>
<th>KT4D (2)</th>
<th>T4D + B4D (3)</th>
<th>2 + 3</th>
<th>1 + 2 + 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{opt} )</td>
<td>56</td>
<td>67</td>
<td>88</td>
<td>80</td>
<td>76</td>
</tr>
</tbody>
</table>

In the light of the obtained results, it is recommended to urgently increase the number of ULF type trams.

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