RISK PRIORITY NUMBER ESTIMATION USING INTUITIONISTIC-FUZZY NUMBERS IN POWER ENGINEERING

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Abstract - This paper considers the evaluation of the Risk Priority Number (RPN) for FMEA approaches. There are presented the traditional RPN method, and existing fuzzy logic based methods. Intuitionistic fuzzy numbers (IFNs) and computational methods involving IFNs are described, and a new methodology for RPN estimation is proposed. Finally, the new IFN-FMEA risk assessment is explained and its usage is shown for the power engineering field.

Keywords: Risk Priority Number, Intuitionistic Fuzzy Numbers, FMEA/FMECA, Risk Management.

1. INTRODUCTION

Maximizing systems dependability and minimizing risk are important objectives of any industrial, financial, or social organization. Recently, risk management in various fields is applied under imprecision data [16, 17, and 28]. The risk estimations are influenced not only by uncertainties but also by imprecision in providing exact evaluation/assessment.

This paper proposes the usage of intuitionistic-fuzzy numbers to evaluate the risk priority numbers used during FMEA procedure, by organizations implementing the continuous improvement in design and exploitation.

2. FMEA: THE BASIC METHODOLOGY

FMEA (Failure Mode and Effects Analysis) is an analysis methodology which has been significantly documented by NASA in 1963 [9, 18, and 24] in order to improve the reliability of specific systems, including software systems [15]. Nowadays, FMEA is a reliability tool applied by a specialized working team, including experts in the field, being able to cover all aspects about the product/system/process under analyse.

FMEA is generally viewed as a nine steps procedure, as shown in fig. 1.

A variant of FMEA is FMECA (Failure Mode, Effects and Criticality Analysis), which include a criticality evaluation based on criticality numbers taking into account the failure effect probability, the failure mode ratio, the component failure rate, and the operating time for each item failure mode [24]. As mentioned in [17], FMECA "have seen wide applications in various domains such as aerospace, nuclear, power generation, petrochemical and other industries".

For a completely analysis, the following aspects should be considered: a) the existence of both single and multiple failure modes; b) the usage of models based on imprecision (subjective probabilities, fuzzy numbers, vague numbers [20], intuitionistic-fuzzy numbers etc.); c) the existence of importance degrees of some parameters; d) the quality of conversion procedures (defuzzification/ crispification algorithms); e) the quality of ranking procedures; f) the necessity of multiple experts participating during evaluation/analysis etc.

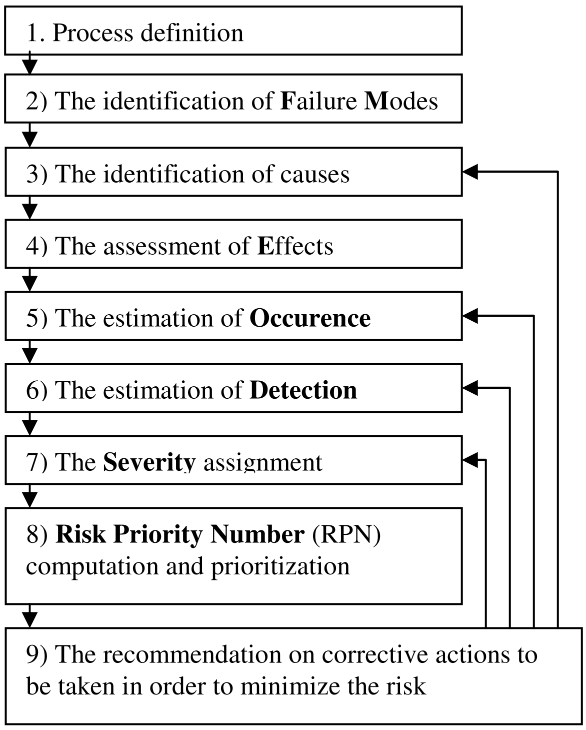

Fig. 1: **FMEA: the basic methodology**

3. RISK PRIORITY EVALUATION

Traditional RPN (Risk Priority Number) is computed by the multiplication of the following parameters: the *severity* **S** (the impact) – a measure indicating the gravity of the effects of a failure/hazard which affect the whole system or a vital component, the *occurrence* **O** – a measure indicating the probability of occuring a failure or a hazard, and the *detection* **D** – a measure indicating the detectability of the failure/hazard by adequate methods of control or inspections: RPN = *severity* x *occurrence* x *detection*. In this manner, the RPN defines the priority of the failure and it is used to rank the potential deficiencies. This is an important step during

FMEA (Failure Mode and Effects Analysis) procedure. Various methods to compute RPN are available [7-14, 18, 23, 24], and [25]. The classical approach assumes that S, O, and D are defined by natural numbers among 1 and 10 (see [23, 24]), where 10 indicates the fault/hazard that is the most severe or the highest occurrence frequency or the most difficult to detect. It is easy to see that product SxOxD results in less numbers (120 effectively) than 1000, some results having a high repetition. A simple example illustrates the repetition: 2x10x10 = 10x2x10 = 10x10x2 = 200. To avoid the repetition, the practitionars considered a large plethora of modifications: from including suplimentary information (like cost) to the usage of fuzzy or vague sets.

Also the traditional RPN considers the same importance of the three parameters: *the three failure mode indexes are all equally important*. In [18] it is proposed the usage og the following weights: 0.5396 (for S), 0.2970 (for O), and 0.1634 (for D). Also, Arabian-Hoseynabady, Oraee and Tavner, used during RELIAWIND project, the following weigts: (0.21, 0.26, 0.53). Other variants are given by [25]. This is to show that the importance degree depends on the system under study by the FMEA methodology.

Another problem with RPN interpretation is related to the assumption that the scales of the three S , O and D indexes have the same metric and that the same danger level corresponds to the same values on different index scales.

This paper proposes the usage of intuititionistic-fuzzy numbers (mainly triangular, but other shapes can be used) to obtain the IF-RPN. Every parameter is given by linguistic variabiles modelled by intuitionistic-fuzzy numbers. The adequate operations will be used to compute the IF-RPN shapes. For rare events (low occurrence frequency), when the probability is difficult to estimate, a subjective probability (provided by experts) can be used. However, subjective probabilities differ from person to person. Because the probability is subjective, it contains a high degree of personal bias, and a multi-expert approach is necessary to be applied.

4. INTUITIONISTIC-FUZZY NUMBERS

According to [2, 3, 4, 5, 6], an intuitionistic fuzzy (IF) set A in a nonempty set X is an object having the form A = {(x, μ(x), ν(x)), x in X}, where μ(x), (respective ν(x)) is the membership (respective non membership) degree. If $\mu(x) + \nu(x) = 1$, for all x in X, then A is a fuzzy set (as introduced by Zadeh [27]; see also [21] for fuzzy arithmetic computational details). If there is at least one element x of X such that $\mu(x) + \nu(x) < 1$ then A is an intuitionistic fuzzy set, or Atanassov set.

The basic concepts of intuitionistic fuzzy modelling are related to operations on intuitionistic fuzzy set, intuitionistic fuzzy relations, intuitionistic fuzzy numbers, and intuitionistic fuzzy intervals.

If {A_i; i in I} are a collection of intuitionistic fuzzy sets over X, then A1 $\cup$ A2 $\cup$ … = {(x, inf ($\mu_1(x)$, $\mu_2(x)$, …), sup ($\nu_1(x)$, $\nu_2(x)$, …); x in X}, where A_i is {(x, $\mu_i(x)$, $\nu_i(x)$)). In a similar way, A1 $\cap$ A2 $\cap$ … = {(x, sup ($\mu_1(x)$, $\mu_2(x)$, …), inf ($\nu_1(x)$, $\nu_2(x)$, …)}. Two intuitionistic fuzzy sets are equal, if their membership, and non membership functions are, corespondingly, equal ($A_1 = A_2$ if and only if $\mu_1(x) = \mu_2(x)$, and $\nu_1(x) = \nu_2(x)$, for all x in X). Similarly, $A_1 \subset A_2$ if and only if $\mu_1(x) \le \mu_2(x)$, and $\nu_1(x) \ge \nu_2(x)$, for all x in X.

An intuitionistic fuzzy relation in X x Y is an intuitionistic fuzzy set of X x Y: R = {((x, y), μ_R(x, y), ν_R(x, y)), (x, y) in X x Y}.

Let R (respective S) be intuitionistic fuzzy relation of X x Y (respective of Y x Z). The composition of R and S is given by $\mu_{R\circ S}$(x, z) = sup {inf (μ_R(x, y), μ_S(y,z)), y in Y}, and $\nu_{R\circ S}$(x, z) = inf {sup (ν_R(x, y), ν_S(y,z)); y in Y}, for all (x, z) in X x Z.

For any (λ, θ) such that $0 \le \lambda + \theta \le 1$, the ($\lambda$, θ) – cut set of A is given by A(λ, θ) = {x | $\mu(x) \ge \lambda$, and $\nu(x) \le \theta$}. Cutting is a very useful process and helps in proving various facts on intuitionistic fuzzy set environment.

The intuitionistic fuzzy numbers (IFNs)are defined over the real line, based on the following elements:

(1) a convex membership function: $\mu(\lambda x_1+(1-\lambda)x_2) \ge \inf(\mu(x_1), \mu(x_2))$,

(2) a concave non membership function: $\nu(\lambda x_1+(1-\lambda)x_2) \le \sup(\nu(x_1), \nu(x_2))$, and

(3) there is x_0 and x_1, real numbers, such that $\mu(x_0) =1$ and $\nu(x_1) = 0$.

In practice, triangular (TIFN) and trapezoidal (TrIFN) intuitionistic fuzzy numbers are mostly popular. In this paper triangular intuitionistic fuzzy numbers will be used. The TIFN A is described by five real numbers *(m; a, b; a', b'), a'* > a, and *b'* > *b* (with *a, b; a', b'* as positive distances around *m*), and two triangular functions

$$\mu_A(x) = \begin{cases} \dfrac{x-m+a}{a}, & \text{for m-a} \le x \le m \\ \dfrac{b+m-x}{b}, & \text{for m} \le x \le m+b \\ 0, \text{ otherwise,} \end{cases}$$

and

$$\nu_A(x) = \begin{cases} \dfrac{m-x}{a'}, & \text{for m-}a' \le x \le m \\ \dfrac{x-m}{b'}, & \text{for m} \le x \le m+b' \\ 1, \text{ otherwise.} \end{cases}$$

Let be the TIFN α= (m; a, b; a', b') where a' > a, and b' > b. The number α is positive if m- a'>0. To fulfill the aim of this paper we need the following properties (that can be proved using the method based on cuts [22]):

1. [*Positive scalar multiplication*] If TIFN α = (m; a, b; a', b') and k > 0 (a positive scalar), then the TIFN kα is given by (km; ka, kb; ka', kb').
2. [*Negative scalar multiplication*] If TIFN α = (m; a, b; a', b') and k < 0 (a negative scalar), then the TIFN kα is given by (km; kb, ka; kb', ka').
3. [*Addition*] If $\alpha = (m_1; a_1, b_1; a_1', b_1')$ and $\beta = (m_2; a_2, b_2; a_2', b_2')$ are TIFNs, then the sequence defined by

(m_1+m_2; a_1+a_2, b_1+b_2; $a_1'+a_2'$, $b_1'+b_2'$) describes the TIFN, $\alpha\oplus\beta$, i.e. the sum of α and β.

4. [*Multiplication*] If $\alpha = (m_1; a_1, b_1; a_1', b_1')$ and $\beta = (m_2; a_2, b_2; a_2', b_2')$ are TIFNs, then the sequence defined by (m_1m_2; a_1a_2, b_1b_2; $a_1'a_2'$, $b_1'b_2'$) describes the TIFN $\alpha\otimes\beta$.

In order to compare two triangular intuitionistic fuzzy numbers some methods are described in literature [26]. Also it is possible to compare TIFNs using the distance measure proposed in [19] and the COA-crispification procedure developed by Angelov [1] and based on Center of Area.

If $\alpha = (m_1; a_1, b_1; a_1', b_1')$ and $\beta = (m_2; a_2, b_2; a_2', b_2')$ are two TIFNs, then $d(\alpha, \beta)$ as one TIFN object is given by $(m; a, b; a', b')$ where $a = (m_1 - m_2) - max\ \{0, (m_1 - m_2) + (a_1 + b_2)/2\}$, $a' = (m_1 - m_2) - max\ \{0, (m_1 - m_2) + (a_1' + b_2')/2\}$, $b = (a_2+b_1)/2$, $b' = (a_2'+b_1')/2$, and m is computed by the *1-cut* method [19].

In this paper we propose the usage of a simple COG-crispification procedure (base on **C**enter **o**f **G**ravity): Let T_1 and T_2 be two TIFNs given by (μ_1, ν_1) and (μ_2, ν_2), respectively. The two planar curves (μ_i, ν_i) generate two 4-sided polygons P_i (i in $\{1, 2\}$). Let t_i be the abscise of the Center of Gravity of the polygon P_i. We define T_1 **LE** T_2 if and only if $t_1 \leq t_2$ (**LE** : **L**ess or **E**qual). This approach can be easily extended to intuitionistic fuzzy trapezoidal numbers.

5. THE IFN – FMEA PROCEDURE

The FMEA procedure based on IFN-RPN is developed taking into account the following rules:

1. [*The subjective probability variant*] Given S(s; s_1, s_2, s'_1, s'_2) the *severity model* as TIFN, given p in [0, 1], the (subjective) *occurrence probability* of the failure, and given D(d; d_1, d_2, d'_1, d'_2) the *detectability index*, as TIFN, then the TIFN-RPN result, denoted by T, is: $pS\otimes D$, where $\otimes$ is the multiplication operator, introduced above. The positive scalar multiplication is also used.
2. [*The full IFN variant*] Given S(s; s_1, s_2, s'_1, s'_2) the Severity model as TIFN, given the Occurrence index O as TIFN(p; p_1, p_2, p'_1, p'_2), and given D(d; d_1, d_2, d'_1, d'_2) the Detectability index, as TIFN, then the TIFN-RPN result, denoted by T, is: $S\otimes O\otimes D$, where $\otimes$ is the IFN multiplication operator.

In order to describe the IFN-FMEA methodology, let be identified the following elements: n – the number of failures under analysis, F_i (i in {1, 2, …, n}) the i^th failure described by (S_i, p_i, D_i) or (S_i, O_i, D_i) depending on the variant selected initially, LE is a sorting operator (defined by COA or COG, or taking into account a specific metric), and φ - a defuzzification/crispification procedure. The IFN-FMEA will consist of the three steps, namely:

1. **IFN-RPN computation**: $T_i = p_iS_i\otimes D_i$ (the first rule), or $T_i = S_i\otimes O_i\otimes D_i$ (the second rule), i in {1, 2, …, n};

2. **Ranking**: Rank the failures according to the LE relation applied on the T_i sequence of TIFN-RPNs, where $t_i = \varphi(T_i)$, i in {1, 2, …, n}, in the case of COG method, and

3. **Take corrective measures/actions** as for usual FMEA.

It is expected a better behaviour of TIFN-FMEA due to the existence of both a membership and a non-membership function, and the availability of many variants in selected the ranking operator. Mainly, the proposed approach is better then the simple fuzzy technique because the region is a 4-point polygon in the case of TIFN, while for fuzzy numbers, the region is a triangle. In the first case, the centroid of TIFN depends also on the non-membership function. A better behaviour is expected for trapezoidal intuitionistic fuzzy numbers (fig. 2).

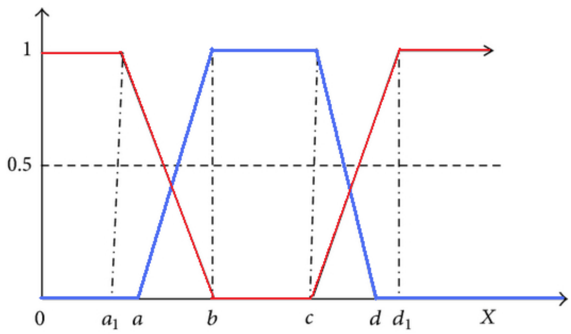

Fig.2. A trapezoidal intuitionistic fuzzy number

Moreover, the model (TIFN-RPN, LE) can solve the case when the same traditional RPN is obtained for situations characterized by different danger levels.

6. WIND TURBINE FMEA

According to [16], FMEA has been extensively used by wind turbine assembly manufactures in order to prioritize the potential failure modes, and realize risk and reliability analysis. Examples of failure modes are: fatigue fracture (the most common), material deformation, misalignment etc, with typical causes like: over stressing, overheating, assembly error, callibration error, maintenance fault etc. The detection of failure modes is done through visual inspection, monitoring techniques, and time-based preventice maintenance actions. The results of failure modes, the effects, are oriented toward loss of electricity production, poor power quality to the grid, and a significant audible noise, as documented in [16].

According to [16], FMEA is not so easy to be implemented in offshore wind farms do to unreliable data collected by the SCADA system, imprecise data provided by experts, and the dificulty of prioritize the S, O, and D parameters.

Only four levels were used by RELIAWIND project, cited in [16], to evaluate the severity (S in {1, 2, 3, 4}, Table 1), occurrence (O in {1, 2, 3, 5}, Table 2), and the failure detectability (D in {1, 4, 7, 10}, Table 3), that means 64 combinations, with only 39 different RPNs. The RELIAWIND team considered the following importance weights (0.21, 0.26, 0.53) to (S, O, D), with more importance for the detectability index.

Table 1. S-levels for wind turbine FMEA [16]

S	Linguistic Variable	Criteria
1	Minor	Urgent repair is necessary even electricity can be generated.
2	Marginal	Reduction in ability to generate electricity is observed.
3	Critical	Loss of ability to generate electricity
4	Catastrophic	Major demage to the installation.

Table 2. O-levels for wind turbine FMEA [16]

O	Linguistic Variable	Criteria
1	Extremely unlikely	A single failure mode probability of occurrence is less than 10^{-3}.
2	Remote	A single failure mode probability of occurrence is more than 10^{-3}, but less than 10^{-2}.
3	Occasional	A single failure mode probability of occurrence is more than 10^{-2}, but less than 10^{-1}.
5	Frequent	A single failure mode probability of occurrence is greater than 10^{-1}.

Table 3. D-levels for wind turbine FMEA [16]

D	Linguistic Variable	Criteria
1	Almost sure	Current methods used to detect the failure modes always will detect the failure.
4	High	There is a great confidence that the current methods will detect the failure.
7	Low	There is a low confidence that the current methods will detect the failure.
10	Almost impossible	No known methods are available to detect the failure.

7. SOFT COMPUTING DETAILS

The TIFN (or TrIFNs) can be used to solve an IFN-FMEA problem for a set of failures under sorting according to the level of IFN-RPNs, as proposed above. In the following the IFN-FMEA approach is shown starting from considerations given in [16], where the system under study belongs to the field of Offshore Wind Energy, as described above.

Using TIFN to model the S/O/D levels described above and updated, the following selections (tables 4, 5, and 6) proved to be suitable for wind turbine TIFN-FMEA. The completely TIFN-FMEA shown similar results like those reported in [16] and obtained by fuzzy rules.

Table 4. TIFN models for S-levels (S in {1, 4, 7, 9})

Linguistic variable	Description	TIFN-FMEA model: (m; a, b, a', b')
Minor	Electricity can be generated but urgent repair is required	(**1**; 0.1, 0.1, 0.2, 0.2)
Marginal	Reduction in ability to generate electricity	(**4**; 0.1, 0.1, 0.2, 0.2)
Critical	Loss of ability to generate electricity	(**7**; 0.2, 0.2, 0.3, 0.3)
Catastrophic	Major damage to the Turbine as a capital installation	(**9**; 0.3, 1, 0.4, 1)

Table 5. TIFN models for O-levels:IF/Subjective cases

Linguistic variable	Description	TIFN-FMEA model: (m; a, b, a', b'), **p** - the crisp/subjective value
Extremely rare	A single failure mode probability of occurrence is less than 0.001	(0.0005, 0.0004, 0.0005, 0.0005, 0.0005); p = **0.0004**
Occasionally	A single failure mode probability of occurrence is more than 0.001 but less than 0.01	(0.007; 0.006, 0.003, 0.007, 0.003); p = **0.0055**
Frequent	A single failure mode probability of occurrence is more than 0.01 but less than 0.10	(0.05; 0.04, 0.05, 0.05, 0.05); p = **0.055**
Unpleasant	A single failure mode probability of occurrence is greater than 0.10	(0.5; 0.4, 0.5, 0.5, 0.5); p = **0.45**

Table 6. TIFN models for D-levels (D in {2, 5, 7, 10})

Linguistic variable	Description [16]	TIFN-FMEA model: (m; a, b, a', b')
Almost sure	Current monitoring methods almost always will detect the failure	(**2**; 1.9, 1, 2, 4)
High	Good likelihood current monitoring methods will detect the failure	(**5**; 2, 1, 3, 2); see fig. 3.
Low	Low likelihood current monitoring methods will detect the failure	(**7**; 1, 2, 2, 2)
Almost impossible	No known monitoring methods available to detect the failure	(**10**; 1, 0, 2, 0)

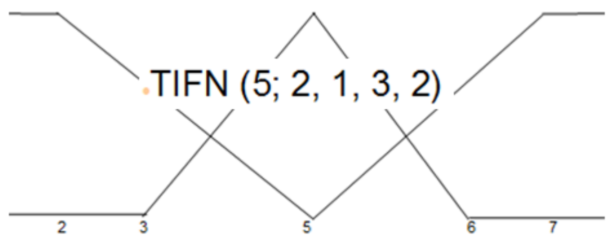

Fig. 3. Graphical representation of TIFN (5; 2, 1, 3, 2)

Computing example (one rule from an IFN-Base Rule System for wind turbine FMEA): *If the Severity is* **Marginal**, *the failure appears* **Occasionally**, *and the Detectability is* **Low** *then* TIFN-RPN = T. T = ?

According to the above tables, and using the first computing rule with p = 0.0055, then **T** = (0.154, 0.0253, 0.04895, 0.0495, 0.0539), as shown in fig. 4, and corresponds to a crisp value t = 0.156228.

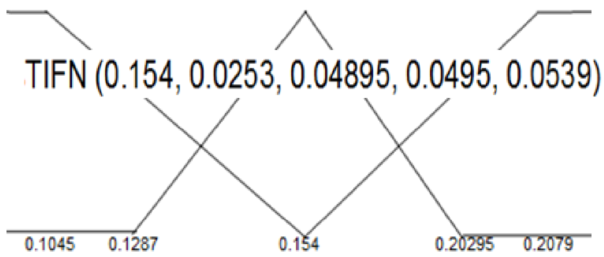

Fig. 4. Graphical representation of T

Using the proposed approach and selecting appropriate models (TIFN, TrIFN) and crispification methods, the failures can be ranked without ambiguity. This is a clear improvement of the clasical FMEA, but uses more computational operations.

8. CONCLUSION

This paper has described the usage of intuitionistic-fuzzy numbers to evaluate the risk priority numbers in order to avoid ambiguity and to facilitate a better ranking of failures/hazards when used for risk and reliability analysis..

Computational details (intuitionistic-fuzzy arithmetics: positive and negative scalar multiplication, addition, and multiplication operators) and sorting algorithms are formulated in an IF – environment.

Finally a case study are used to demonstrate some computational details of the proposed approach, to extend the discussion on „A Fuzzy-FMEA risk assessment approach for offshore wind turbines“, available in [16].

Commuting from the discrete scale to intuitionistic-fuzzy modelling offers to the specialist/expert more freedom to appreciate the required level (of severity, occurrence, and detectability). Even the proposal is a general one, it may be useful to many fields of activity (mainly for risk management department).

The future developments are dedicated to:

- the usage of intuitionistic-fuzzy intervals and their arithmetic to compute RPNs as intervals and update the IF-FMEA procedure;
- the development of an expert system for FMEA/FMECA approaches based on intuitionistic-fuzzy entitities (numbers, intervals, union of numbers and intervals).

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